

Pricing Communication Networks: Optimality and Incentives

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*Dedicated to **Mom, Dad, and Robert,**
for your endless love and support.*

Abstract

Network pricing is a cross-disciplinary research area, which requires deep understanding of both networking technology and microeconomics. The goal of network pricing is to achieve satisfied network performances by allocating the scarce resource to satisfy different users' qualities of services while keeping in mind the incentives of different network entities. Proper design of pricing schemes is indispensable to the operation and management of communication networks. In this thesis we divide network pricing into four categories: static optimization-oriented pricing, dynamic optimization-oriented pricing, static profit-driven pricing, and dynamic profit-driven pricing. The first one is well studied in the literature, and our focus will be on the latter three categories. For each category, we illustrate the key design challenges and insights through a concrete networking example.

First, we investigate the issue of static profit-driven pricing. We consider a revenue maximization problem for a monopolist service provider, and discuss how to set incentive-compatible prices to induce proper allocation of limited resources among different types of users. We capture the interaction between the service provider and users through a two-stage Stackelberg game with both complete and incomplete information. With complete information, we study three pricing schemes: complete price differentiation, partial price differentiation, and no price differentiation. We characterize the trade-offs between the performance and com-

plexity of different schemes. With incomplete information, we show that it is still possible to realize price differentiation, and provide the sufficient and necessary condition under which an incentive compatible price differentiation scheme can achieve the same revenue as the best scheme with complete information.

Then we investigate the issue of dynamic profit-driven pricing. We consider a general resource allocation and profit maximization problem for a cognitive virtual mobile network operator. Dynamics of the cognitive radio network include dynamic user demands, unstable sensing spectrum resources, dynamic spectrum prices, and time-varying channel conditions. In addition, we also consider multi-user diversity and imperfect sensing technique so that the network model is more realistic. We develop a low-complexity on-line control policy that determines pricing and resource scheduling without knowing the statistics of dynamic network parameters. We show that the proposed algorithm with dynamic pricing can achieve arbitrarily close to the optimal profit with a proper trade-off with the queuing delay.

We later investigate the issue of dynamic optimization-oriented pricing. We consider a node-capacitated multicast network with time-varying topology. By utilizing network coding, we design a dynamic pricing scheme that can achieve arbitrarily close to maximum network utility in a distributed fashion, while maintaining network stability. Moreover, we show that this algorithm is incentive-compatible, *i.e.*, no matter what role a node plays in the network, the algorithm guarantees that the node has a non-negative profit. This result has practical importance for constructions for node-capacitated networks with multiple individual users (*e.g.*, P2P networks), since it provides the proper incentives for individual nodes to join, stay, and contribute as relays in the network even if they have no interested contents.

The results developed in this thesis highlight the importance of pricing in communication networks. Specifically, our results show that pricing can be used as an effective tool to achieve optimal network performances while providing proper incentives for all network entities. This not only helps us better understand network pricing, but also gives us insights on the design of network pricing schemes.

摘要

網絡定價是一個基于對網絡技術和微觀經濟學深刻理解而產生和發展的交叉學科。其目標在於通過合理分配稀缺的網絡資源以滿足不同用戶的服務質量，同時又兼顧考慮對網絡中各個不同實體的相應激勵，從而實現令人滿意的網絡性能。適宜的定價設計在通訊網絡的運營和管理中都是必不可缺的。在本論文中，我們將網絡定價分為四類：面向優化的靜態定價、面向優化的動態定價、利潤驅動的靜態定價，和利潤驅動的動態定價。第一類定價問題已經在文獻中深入討論，本論文將集中討論後三類定價問題。對於每一類定價問題，我們將通過一個網絡定價設計實例來闡明定價設計中的關鍵挑戰與深刻見解。

首先，我們研究了利潤驅動的靜態定價。我們考慮了一個壟斷型的網絡運營商的利潤最大化問題，討論其如何設計激勵相容的價格，從而使得有限的網絡資源在不同類型用戶間合理地分配。我們通過完全信息和非完全信息下的雙層斯塔伯格博弈模型來建模刻畫運營商和用戶之間的相互作用。在完全信息下，我們研究了三種定價策略：完全價格分化、部分價格分化，和無價格分化。我們分析了這些不同定價策略在系統性能和複雜度之間的權衡關係。在不完全信息下，我們展示了設計價格分化策略的可能性，並且給出這種激勵相容的定價策略能使運營商獲得完全信息下價格分化定價策略所獲得之相同收益的充分必要條件。

接著，我們研究了利潤驅動的動態定價。我們考慮了一個認知網絡虛擬移動網絡運營商的資源分配與利潤最大化的一般問題。認知網絡的動態性包括動態的用戶

需求、不穩定的檢測頻譜資源、動態的頻譜租用價格，以及時變的無線信道條件。另外，為使網絡模型更接近於實際，我們還考慮了多用戶差異性和有缺陷的頻譜檢測技術。我們設計和發展了一套低複雜度的在線控制策略，能夠在不知動態網絡參數的統計特性的情況下，確定定價和資源分配。我們證明這套動態定價的算法在適當權衡網絡延時的條件下，可以無限趨近最大利潤。

最後，我們研究了面向優化的動態定價。我們考慮一個節點容量受限的拓撲時變的多播網絡。通過運用網絡編碼，我們設計了一套動態定價策略可以分佈式地實現無限趨近最優的網絡性能。另外，我們證明這套算法是激勵相容的，即無論節點在網絡中充當任何角色，該算法都可以保證該節點獲得非負的收益。這個結果表明，該算法可以給網絡節點提供有效的激勵，使之加入網絡、停留在網絡中，並且即使在沒有自身感興趣內容時，也願意充當其他節點的中繼，這一結果在多用戶的節點容量受限網絡（如 P2P 網絡）的構建中有著重要的現實意義。

以上本論文推導之結果都展示了網絡定價在通訊網絡中的重要意義。尤其顯示了網絡定價是實現最優網絡性能，同時對各網絡實體提供激勵的有效工具。本論文不僅幫助我們更好地理解網絡定價問題，同時也給出網絡定價設計中的深刻見解。

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Chapter 1

Introduction

Pricing plays an important role for the design, operation, and management of communication networks. Traditionally, engineers have designed communication services without worrying about how these services should be priced. This may be due to the fact that in early time, there are very few monopolist service providers in the market provisioning very few services (mainly voice service and a little bit data service). These service providers are guaranteed enough profit, even with simple flat-fee pricing schemes *e.g.*, unlimited data plans that have dominated the communication market for quite a long time.

However, the recent technology advances bring major changes to the communication industry. On one hand, the demands for both wireline and wireless data services have been growing exponentially due to the increasing popularity of Internet multimedia applications, mobile devices, and social networks. Statistics from Cisco's white paper [1,2] show that "Global IP traffic has increased eightfold over the past 5 years, and will increase fourfold over the next 5 years. In 2015, global IP traffic will reach 966 exabytes per year or 80.5 exabytes per month", "Global mobile data traffic will increase 18-fold between 2011 and 2016. Mobile data

traffic will grow at a compound annual growth rate (CAGR) of 78 percent from 2011 to 2016, reaching 10.8 exabytes per month by 2016”. On the other hand, the expansion of the market has spawned a highly competitive environment for service providers. For example, the increasingly expensive radio spectrum resources significantly increase the capital and operational costs. In the European 3G spectrum auction, a 20 MHz frequency band costs as high as multi-billion dollars [3]. With no doubt, these major changes in communication networks also lead to major challenges for service providers in making pricing strategies for both wireline and wireless services. Randall Stephenson, chief executive of AT&T, recently commented on the state of the communication industry [4]: “My only regret was how we introduced pricing in the beginning (the flat-fee pricing)”. In fact, these new challenges in communication networks make service providers worldwide, including AT&T, gradually move to more sophisticated pricing schemes. These challenges also necessitate the emergence of a cross-discipline academic research of “network economics”¹, which searches for new approaches to the modeling and management of communication networks.

In this thesis, we study one key issue in the network economics: how to design pricing schemes for communication networks. There are multiple dimensions of the network pricing. According to different network scenarios, there are static pricing schemes for deterministic networks, and dynamic pricing for stochastic networks. In terms of the functionalities of network pricing, it is not only useful for compensating costs and making profits, but also is an effective tool for the net-

¹Network economics was initiated for pricing Internet traffic (wireline). The recent trend shows that wireless data traffic has been growing much faster than the wireline traffic. It is expected that traffic from wireless devices will exceed traffic from wireline devices by 2015 [1]. Thus a lot of research activities on network economics now happen in the wireless domain.

work operation and management. By exploring different dimensions of network pricing, this thesis study modeling, analysis, and algorithm design for a wide range of communication networks. We consider different perspectives of network participants including the end users, the service providers, and the social planners; we consider both deterministic network and the stochastic network; we also consider both centralized and distributed algorithm design. Optimality and incentives are two major recurring themes through this network pricing study. On one hand, no matter which role it plays in a network, a selfish end-user, a profit-driven operator, or a social planner, the optimality of its own objective is always a desirable goal or at least a benchmark for the related pricing problem. On the other hand, since communication markets include multiple participants (*e.g.*, service providers and millions of individual users), we should take the incentive issues into consideration, as strategic behaviors of different participants have big impacts on the pricing and resource allocation result.

In this chapter, we will give a high-level overview of this thesis, and introduce the preliminary backgrounds and methodologies for the network pricing study. In Section 1.1, we review the pricing schemes in communication networks. Section 1.2 provides a brief introduction for two main mathematical tools used in the network pricing study, which serves a pivotal role in solving the pricing related optimization problem in this thesis. The main results of this thesis are outlined in Section 1.3.

1.1 Pricing Schemes in Communication Networks

In the area of communication networks, existing pricing schemes can be classified into two main categories: optimization-oriented pricing schemes and profit-driven

pricing schemes.

Optimization-oriented pricing schemes in communication networks are initiated by MacKie-Mason and Varian [5], and further developed in Kelly's work on network congestion control [6, 7]. Following this line of work, the Transmission Control Protocol (TCP) has been successfully reverse-engineered as a congestion pricing based solution to a network optimization problem [8, 9]. A more general framework of Network Utility Maximization (NUM) was subsequently developed to forward-engineer many new network protocols (see a recent survey in [10]). In various NUM formulations, the "optimization-oriented" prices often represent the Lagrangian multipliers of various resource constraints and are used to coordinate different network entities to achieve the maximum system performance in a distributed fashion.

From an economic perspective, optimization-oriented pricing schemes are associated with social welfare maximization problems. They focus on network protocol design and individual entity based distributed implementation. The prices here are devised by a virtual social planner as the "invisible hand". However, most commercial communication networks today are built and managed by service providers with profit maximization in mind. Therefore, service providers in practical network operation are more likely to adopt the profit-driven pricing schemes. These schemes provide incentive ways for service providers to set prices that they are ready to offer and their customers are willing to accept. During the past decades, service providers have preferred to using simple pricing schemes, *e.g.*, the flat-fee pricing. But today, with an exponential growth of data volume and applications in both wireline and wireless networks, service providers are gradually shifting to more sophisticated profit-driven pricing schemes.

Based on specific different network scenarios, each of the pricing category

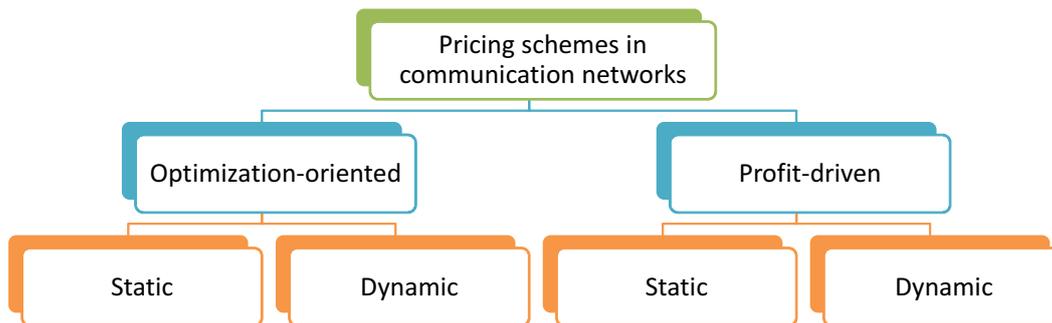


Figure 1.1: Taxonomy of pricing schemes in communication networks

can be further divided into two sub-categories: static pricing and dynamic pricing. A taxonomy of pricing schemes in communication networks can be found in Figure 1.1. Now, in the bottom row of this taxonomy in Figure 1.1, we have four categories of pricing schemes. The first category, static optimization-oriented pricing schemes, has been thoroughly studied in the NUM framework during the past decade [10]. Therefore, this thesis omits the first category and focuses on the other three categories.

1.2 Two Main Algorithm Design Techniques

1.2.1 Network Utility Maximization

In the past decade, we have witnessed the development of Network Utility Maximization (NUM) framework, which provides a systematic way for network protocol analysis and design. The central idea is to associate each individual network entity with a utility function, and to view the network protocol as a distributed solution to the global optimization problem of maximizing aggregate entities' utilities. In this way, people have successfully reverse-engineered many commonly used network protocols (*e.g.*, TCP/IP [8, 9, 11–15], BGP [16], contention-based

MAC protocols [17, 18]). With the insights obtained from the reverse-engineering work, forward-engineering studies further systematically improve the existing network protocols.

A typical form of the NUM problem is given as follows:

$$\begin{aligned} & \text{Maximize} && \sum_{i \in \mathcal{I}} U_i(x_i) && (1.1) \\ & \text{subject to} && \mathbf{R} \mathbf{x} \leq \mathbf{c} \\ & \text{variables} && \mathbf{x} \end{aligned}$$

where x_i represents the resource allocated to entity i and $U_i(\cdot)$ denotes the utility function of entity i . The utility functions are usually interpreted as the entities' preferences of the resource allocation, or the measurements of the efficiency and fairness of algorithms. The concrete form of utility functions depends on the protocol design goal. To obtain analytical results, we often assume the utility function to be twice differentiable, increasing and strictly concave. In doing so, the NUM problem is a convex optimization problem. A commonly used family of utility functions is the following α -fairness utility function [13]:

$$U^\alpha(x) = \begin{cases} \frac{x^{1-\alpha}}{1-\alpha} & \alpha \neq 1 \\ \log x & \text{Otherwise} \end{cases}, \quad (1.2)$$

where $\alpha = 1$ corresponds to proportional fairness and $\alpha \rightarrow \infty$ corresponds to max-min fairness. The inequality $\mathbf{R} \mathbf{x} \leq \mathbf{c}$ is a general form of linear constraints in NUM problem, which represents various network constraints (*e.g.*, network connection, channel variation in wireless communication network) and Quality of Service (QoS) requirements (*e.g.*, rate requirement, SINR requirement). For example, when \mathbf{R} denotes the binary network routing matrix, and \mathbf{c} denotes the link capacity vector, then $\mathbf{R} \mathbf{x} \leq \mathbf{c}$ can represent the link capacity constraints.

In protocol design, it is often desirable to have a distributed network algorithm to solve NUM problems, where each network entity makes its own decisions based on local information. From an economic perspective, it is in the same spirit of “invisible hand” described by Adam Smith. The basic idea is proper decomposition: to divide the original large optimization problem into smaller subproblems (for each network entity). Today, we have several readily available decomposition techniques. One most frequently used decomposition technique is based on the Lagrange dual of the original optimization problem. In this dual decomposition method, the Lagrangian multipliers can be interpreted as prices charged to the resource allocation in the subproblems, in which network entities decide the amount of resource to be used according to the prices. Thus the dual decomposition approach can be understood as a pricing mechanism design, which falls into the category of the aforementioned optimization-oriented pricing schemes.

1.2.2 Lyapunov Stochastic Optimization

Lyapunov stochastic optimization technique (an application of which is also known as Max-Weight or Backpressure based scheduling) is the state-of-the-art tool for solving stochastic network optimization problems. It was first proposed in the seminal work [19] for the optimization of wireless networks [20], and was later developed into a general framework for stochastic network optimization [21]. It has been proven to be capable of solving a wide range of communication and queueing network problems [21], for instance, throughput optimal routing, network energy minimization, opportunistic cooperation, dynamic data compression, product-assembly plants, energy allocation for smart grids, inventory control and power reduction for data centers. The Lyapunov optimization technique is currently receiving much attention in both the theoretical and experimental commu-

nities, due to (i) its ability to achieve performance optimality, (ii) its robustness against time-varying and stochastic network conditions, and (iii) its mathematical tractability.

One of the key tools in Lyapunov optimization technique is Lyapunov drift argument [20, 21]. Suppose we are given a discrete-time stochastic network with K queues (possibly including some virtual queues). Let $\mathbf{Q}(t) = (Q_1(t), \dots, Q_K(t))$ be a vector process of queue lengths. We assume that $\mathbf{Q}(t)$ represent network states, which characterize the randomness in the network, such as the network channel condition or the random number of arrivals. $\mathbf{Q}(t)$ is stable if and only if

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n=1}^K \mathbb{E}[Q_n(\tau)] < \infty. \quad (1.3)$$

Let $L(\mathbf{Q})$ be any non-negative scalar valued function of the queue lengths, called a Lyapunov function. Usually we adopt the quadratic Lyapunov function

$$L(\mathbf{Q}(t)) \triangleq \frac{1}{2} \left[\sum_{n=1}^K Q_n^2(t) \right]. \quad (1.4)$$

Now we define the *one-slot conditional Lyapunov drift* $\Delta(\mathbf{Q}(t))$ as follows:

$$\Delta(\mathbf{Q}(t)) \triangleq \mathbb{E}[L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) | \mathbf{Q}(t)]. \quad (1.5)$$

This drift is the expected change in the Lyapunov function over one slot, given that the current state in slot t is $\mathbf{Q}(t)$.

Suppose that, in addition to the queues $\mathbf{Q}(t)$ that we want to stabilize, we have an associated stochastic “penalty” process $f(t)$, whose time average we want to make less than (or close to) some target value f^* . Assume the expected penalty is lower bounded by a finite value 0. We have the following Lyapunov drift theorem:

Theorem 1.1. *Suppose that there exist finite constants $V > 0, B > 0, \epsilon > 0$, and a non-negative function $L(\mathbf{Q})$ such that $\mathbb{E} L(\mathbf{Q}(0)) < \infty$ and for every timeslot*

$\tau \geq 0$, we have:

$$\Delta(\mathbf{Q}(\tau)) + V\mathbb{E}f(\tau)|\mathbf{Q}(\tau) \leq B + Vf^* - \epsilon \sum_{n=1}^K |Q_n(\tau)|, \quad (1.6)$$

then the time average expected penalty and queue length satisfy:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}f(\tau) \leq f^* + \frac{B}{V} \quad (1.7)$$

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n=1}^K \mathbb{E}|Q_n(\tau)| \leq \frac{B + Vf^*}{\epsilon}. \quad (1.8)$$

If for any parameter $V > 0$, we can design a control policy to ensure the drift condition (1.6) is satisfied on every slot τ , then the time average expected penalty satisfies (1.7) and hence it can be arbitrarily close to the target value f^* with a sufficient large value of V . However, according to (1.8), the time average queue length bound increases linearly with V . By Little's Theorem we know that queue length is proportional to average delay, thus we obtain a performance-delay tradeoff of $[O(1/V), O(V)]$. This means when achieving the $O(1/V)$ close-to-optimal performance, one can only guarantee that the incurred network delay is $O(V)$.

The control policy developed using Lyapunov drift argument has many useful features:

- The sequence of base policies are created in a greedy and dynamic fashion, depending on the current network state information. In this regard, we may view it as a learning algorithm over stochastic networks.
- The policy requires limited statistical knowledge of the network, such as arrival rates and channel statistics of the network. In many applications, it does not need any statistical knowledge.
- The resulting policy has a near-optimal performance which can be proved without knowing the exact value of the optimal performance.

More details can be found at [20, 21].

1.3 Thesis Outline

In summary, we divide the results presented in this thesis into three main categories showed in Figure 1.1: static profit-driven pricing schemes, dynamic profit-driven pricing schemes and dynamic optimization-oriented pricing schemes. For each category, we illustrate the key design challenges and insights through a concrete networking example. The rest of this thesis and main results are provided as follows:

Chapter 2 is about static profit-driven pricing. Motivated by the practical needs of industry, in this chapter, we provide a comprehensive study for the usage-based pricing design with different price differentiation levels. Concretely, we consider the revenue-maximizing problem for a monopoly service provider under both complete and incomplete network information. Under complete information, our focus is to investigate the tradeoff between the total revenue and the implementational complexity (measured in the number of pricing choices available for users). Among the three pricing differentiation schemes we proposed (*i.e.*, complete, single, and partial), the partial price differentiation is the most general one and includes the other two as special cases. By exploiting the unique problem structure, we design an algorithm that computes the optimal partial pricing scheme in polynomial time, and numerically quantize the tradeoff between implementational complexity and total revenue. Under incomplete information, it is difficult in general to design an incentive-compatible differentiation pricing scheme. We show that when the users are significantly different, it is possible to design a quantity-based pricing scheme that achieves the same maximum revenue as under

complete information.

Chapter 3 considers dynamic profit-driven pricing. Cognitive radio networks, as one of the most promising solutions to nowadays spectrum shortage problem, have also brought new challenges in future network management. In this chapter, we study a general resource allocation and profit maximization for a cognitive mobile virtual network operator (a mobile virtual network operator equipped with the cognitive radio technique). We consider a downlink OFDM transmission system with various network dynamics, including dynamic user demands, unstable sensing spectrum resources, dynamic spectrum prices, and time-varying channel conditions. In addition, multi-user diversity and imperfect sensing technique are incorporated to make the network model close to the real system. By exploring the special structural information of the problem, we develop a low-complexity on-line control policy that determines pricing and resource scheduling without knowing the network statistics. We show that the proposed algorithm can achieve arbitrarily close to the optimal profit with a proper trade-off with the queuing delay.

Chapter 4 provides an example of dynamic optimization-oriented pricing design for a stochastic network utility maximization problem. We consider a multicast scenario in a time-varying node-capacitated network with network coding. We design a dynamic and distributed pricing and resource allocation algorithm that can achieve arbitrarily close to the optimal network utility (at the expense of large delays) while maintaining the network stability. Moreover, we show this algorithm is incentive-compatible, *i.e.*, no matter what role a node plays in the network, the algorithm guarantees that the node has a non-negative profit. This result has practical importance for distributed network constructions, since it provides the proper incentives for individual nodes to join, stay, and contribute as relays in

the network even if they have no interested contents.

The conclusion to the thesis, Chapter 5, discusses open issues and further research directions.

Chapter 2

Price Differentiation for Communication Networks

We study the optimal usage-based pricing problem in a resource-constrained network with one profit-maximizing service provider and multiple groups of surplus-maximizing users. With the assumption that the service provider knows the utility function of each user (thus complete information), we find that the complete price differentiation scheme can achieve a large revenue gain (e.g., 50%) compared to no price differentiation under two conditions: the total network resource is comparably limited, and the users with high willingness to pay are minorities. However, the complete price differentiation scheme may lead to a high implementational complexity. To trade off the revenue against the implementational complexity, we further study the partial price differentiation scheme, and design a polynomial-time algorithm that can compute the optimal partial differentiation prices. We also consider the incomplete information case where the service provider does not know which group each user belongs to. We show that it is still possible to realize price differentiation under this scenario, and provide the sufficient and necessary

condition under which an incentive compatible differentiation scheme can achieve the same revenue as under complete information.

2.1 Usage-based Pricing Schemes

Economists have proposed many sophisticated pricing mechanisms to extract surpluses from the consumers and maximize revenue (or profits) for the providers. A typical example is the optimal nonlinear pricing [22–24]. In practice, however, we observe simple pricing schemes deployed by the service providers. Typical examples include flat-fee pricing and (piecewise) linear usage-based pricing. One potential reason behind the gap between “theory” and “practice” is that the optimal pricing schemes derived in economics often has a high implementational complexity. Besides a higher maintenance cost, complex pricing schemes are not “customer-friendly” and discourage customers from using the services [25, 26]. Furthermore, achieving the highest possible revenue often with complicated pricing schemes requires knowing the information (identity and preference) of each customer, which can be challenging in large scale communication networks. It is then natural to ask the following two questions:

1. How to design simple pricing schemes to achieve the best tradeoff between complexity and performance?
2. How does the network information structure impact the design of pricing schemes?

This chapter tries to answer the above two questions with some stylized communication network models. Different from some previous work that considered a flat-fee pricing scheme where the payment does not depend on the resource

consumption (*e.g.*, [25, 27, 28]), here we study the revenue maximization problem with the linear usage-based pricing schemes, where a user's total payment is linearly proportional to allocated resource. In wireless communication networks, the usage-based pricing scheme has become increasingly popular due to the rapid growth of wireless data traffic. In June 2010, AT&T in the US switched from the flat-free based pricing (*i.e.*, unlimited data for a fixed fee) to the usage-based pricing schemes for 3G wireless data [29]. Verizon followed up with similar plans in July 2011. Similar usage-based pricing plans have been adopted by major Chinese wireless service providers including China Mobile and China UniCom. Thus, the research on the usage-based pricing is of great practical importance.

In this chapter, we consider the revenue maximization problem of a monopolist service provider facing multiple groups of users. Each user determines its optimal resource demand to maximize the surplus, which is the difference between its utility and payment. The service provider chooses the pricing schemes to maximize his revenue, subject to a limited resource. We consider both complete information and incomplete information scenarios and design different pricing schemes with different implementational complexity levels.

Our main contributions are as follows.

- *Complete network information:* We propose a polynomial algorithm to compute the optimal solution of the partial price differentiation problem, which includes the complete price differentiation scheme and the single pricing scheme as special cases. The optimal solution has a threshold structure, which allocates positive resources with priorities to users with high willingness to pay.
- *Revenue gain under the complete network information:* Compared to the single pricing scheme, we identify the two important factors behind the

revenue increase of the (complete and partial) price differentiation schemes: the differentiation gain and the effective market size. The revenue gain is the most significant when users with high willingness to pay are minority among the whole population and total resource is limited but not too small.

- *Incomplete network information*: We design an incentive-compatible scheme with the goal to achieve the same maximum revenue that can be achieved with the complete information. We find that if the differences of willingness to pay of users are larger than some thresholds, this incentive-compatible scheme can achieve the same maximum revenue. We further characterize the necessary and sufficient condition for the thresholds.

It is interesting to compare our results under the complete network information scenario with results in [25] and [30]. In [25], the authors showed that the revenue gain of price differentiation is small with a flat entry-fee based Paris Metro Pricing (*e.g.*, [31]), and a complicated differentiation strategy may not be worthwhile. Chau et al. [30] further derived the sufficient conditions of congestion functions that guarantee the viability of these Paris Metro Pricing schemes. By contrast, our results show that the revenue gain of price differentiation can be substantial for a usage-based pricing system.

Some recent work of usage-based pricing and revenue management in communication network includes [32–39]. Basar and Srikant in [32] investigated the bandwidth allocation problem in a single link network with the single pricing scheme. Shen and Basar in [33] extended the study to a more general nonlinear pricing case with the incomplete network information scenario. They discussed the single pricing scheme under incomplete information with a continuum distribution of users' types. In contrast, our study on the incomplete information focuses on the linear pricing with a discrete setting of users' types. We also show

that, besides the single pricing scheme, it is also possible to design differentiation pricing schemes under incomplete information. Daoud *et al.* [34] studied an uplink power allocation problem in a CDMA system, where the interference among users are the key constraint instead of the limited total resource considered in this chapter. Jiang *et al.* in [35] and Hande *et al.* in [36] focused on the study of the time-dependent pricing. He and Walrand in [37], Shakkottai and Srikant in [38] and Gajic *et al.* in [39] focused on the interaction between different service providers embodied in the pricing strategies, rather than the design of the pricing mechanism. Besides, none of the related work considered the partial differential pricing as in this chapter.

2.2 System Model

We consider a network with a total amount of S limited resource (which can be in the form of rate, bandwidth, power, time slot, etc.). The resource is allocated by a monopolistic service provider to a set $\mathcal{I} = \{1, \dots, I\}$ of user groups. Each group $i \in \mathcal{I}$ has N_i homogeneous users¹ with the same utility function:

$$u_i(s_i) = \theta_i \ln(1 + s_i), \quad (2.1)$$

where s_i is the allocated resource to one user in group i and θ_i represents the willingness to pay of group i . The logarithmic utility function is commonly used to model the proportionally fair resource allocation in communication networks (see [32] for detailed explanations). The analysis of the complete information case can also be extended to more general utility functions (see Appendix 2.10.1). Without loss of generality, we assume that $\theta_1 > \theta_2 > \dots > \theta_I$. In other words,

¹A special case is $N_i=1$ for each group, i.e., all users in the network are different.

group 1 contains users with the highest valuation, and group I contains users with the lowest valuation.

We consider two types of information structures:

1. **Complete information:** the service provider knows each user's utility function. Though the complete information is a very strong assumption, it is the most frequently studied scenario in the network pricing literature [32–39]. The significance of studying the complete information is two-fold. It serves as the benchmark of practical designs and provides important insights for the incomplete information analysis.
2. **Incomplete information:** the service provider knows the total number of groups I , the number of users in each group $N_i, i \in \mathcal{I}$, and the utility function of each group $u_i, i \in \mathcal{I}$. It does not know which user belongs to which group. Such assumption in our discrete setting is analogous to that the service provider knows only the users' type distribution in a continuum case. Such statistical information can be obtained through long term observations of a stationary user population.

The interaction between the service provider and users can be characterized as a two-stage Stackelberg model shown in Fig. 2.1. The service provider publishes the pricing scheme in Stage 1, and users respond with their demands in Stage 2. The users want to maximize their surpluses by optimizing their demands according to the pricing scheme. The service provider wants to maximize its revenue by setting the right pricing scheme to induce desirable demands from users. Since the service provider has a limited total resource, he must guarantee that the total demand from users is no larger than what he can supply.

The details of pricing schemes depend on the information structure of the ser-

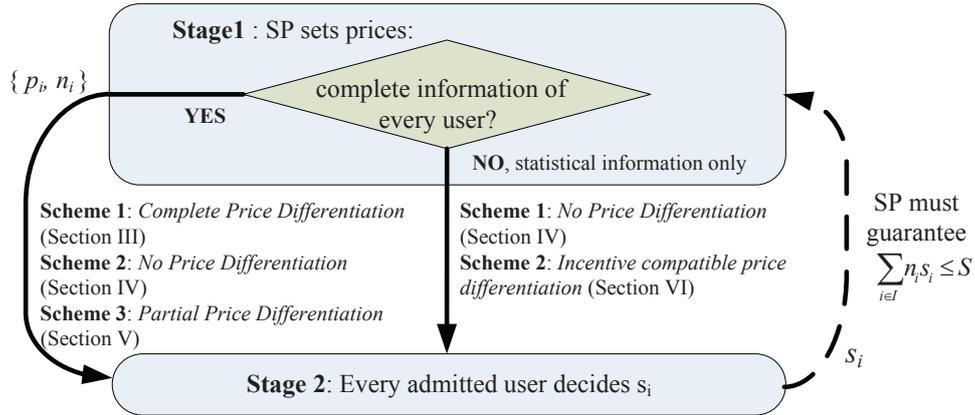


Figure 2.1: A two-stage leader and follower model

vice provider. Under complete information, since the service provider can distinguish different groups of users, he announces the pricing and the admission control decisions to different groups of users. It can choose from the complete price differentiation scheme, the single pricing scheme, and the partial price differentiation scheme to realize desired trade-off between implementational complexity and total revenue. Under incomplete information, it publishes a common price *menu* to all users, and allow users to freely choose a particular price option in this menu. All these pricing schemes will be discussed one by one in the following sections.

Note that it is possible for a user to achieve an “arbitrage” by splitting into several smaller users, each of which requests a small amount of resource and enjoys a lower unit price. Fortunately, preventing arbitrage of services is often easier and less costly than that of goods [40]. For example, we can often uniquely identify a user through its MAC address. Full discussion of arbitrage prevention, however, is beyond the scope of this chapter.

2.3 Complete Price Differentiation under complete information

We first consider the complete information case. Since the service provider knows the utility and the identity of each user, it is possible to maximize the revenue by charging a different price to each group of users. The analysis will be based on backward induction, starting from Stage 2 and then moving to Stage 1.

2.3.1 User's Surplus Maximization Problem in Stage 2

If a user in group i has been admitted into the network and offered a linear price p_i in Stage 1, then it solves the following surplus maximization problem,

$$\underset{s_i \geq 0}{\text{maximize}} \quad u_i(s_i) - p_i s_i, \quad (2.2)$$

which leads to the following unique optimal demand

$$s_i(p_i) = \left(\frac{\theta_i}{p_i} - 1 \right)^+, \quad \text{where } (\cdot)^+ \triangleq \max(\cdot, 0). \quad (2.3)$$

Remark 2.1. *The analysis of the Stage 2 user surplus maximization problem is the same for all pricing schemes. The result in (2.3) will be also used in Sections 2.4, 2.5 and 2.6.*

2.3.2 Service Provider's Pricing and Admission Control Problem in Stage 1

In Stage 1, the service provider maximizes its revenue by choosing the price p_i and the admitted user number n_i for each group i subject to the limited total resource S . The key idea is to perform a Complete Price differentiation (*CP*) scheme, i.e.,

charging each group with a different price.

$$CP : \underset{\mathbf{p} \geq 0, \mathbf{s} \geq 0, \mathbf{n}}{\text{maximize}} \sum_{i \in \mathcal{I}} n_i p_i s_i \quad (2.4)$$

$$\text{subject to } s_i = \left(\frac{\theta_i}{p_i} - 1 \right)^+, \quad i \in \mathcal{I}, \quad (2.5)$$

$$n_i \in \{0, \dots, N_i\}, \quad i \in \mathcal{I}, \quad (2.6)$$

$$\sum_{i \in \mathcal{I}} n_i s_i \leq S. \quad (2.7)$$

where $\mathbf{p} \triangleq \{p_i, i \in \mathcal{I}\}$, $\mathbf{s} \triangleq \{s_i, i \in \mathcal{I}\}$, and $\mathbf{n} \triangleq \{n_i, i \in \mathcal{I}\}$. We use bold symbols to denote vectors in the sequel. Constraint (2.5) is the solution of the Stage 2 user surplus maximization problem in (2.3). Constraint (2.6) denotes the admission control decision, and constraint (2.7) represents the total limited resource in the network.

The CP problem is not straightforward to solve, since it is a non-convex optimization problem with a non-convex objective function (summation of products of n_i and p_i), a coupled constraint (2.7), and integer variables \mathbf{n} . However, it is possible to convert it into an equivalent convex formulation through a series of transformations, and thus the problem can be solved efficiently.

First, we can remove the $(\cdot)^+$ sign in constraint (2.5) by realizing the fact that there is no need to set p_i higher than θ_i for users in group i ; users in group i already demand zero resource and generate zero revenue when $p_i = \theta_i$. This means that we can rewrite constraint (2.5) as

$$p_i = \frac{\theta_i}{s_i + 1} \text{ and } s_i \geq 0, i \in \mathcal{I}. \quad (2.8)$$

Plugging (2.8) into (2.4), then the objective function becomes $\sum_{i \in \mathcal{I}} n_i \frac{\theta_i s_i}{s_i + 1}$. We can further decompose the CP problem in the following two subproblems:

1. *Resource allocation*: for a fixed admission control decision \mathbf{n} , solve for the

optimal resource allocation \mathbf{s} .

$$\begin{aligned} CP_1 : \quad & \underset{\mathbf{s} \geq 0}{\text{maximize}} && \sum_{i \in \mathcal{I}} n_i \frac{\theta_i s_i}{s_i + 1} \\ & \text{subject to} && \sum_{i \in \mathcal{I}} n_i s_i \leq S. \end{aligned} \quad (2.9)$$

Denote the solution of CP_1 as $\mathbf{s}^* = (s_i^*(\mathbf{n}), \forall i \in \mathcal{I})$. We further maximize the revenue of the integer admission control variables \mathbf{n} .

2. *Admission control:*

$$\begin{aligned} CP_2 : \quad & \underset{\mathbf{n}}{\text{maximize}} && \sum_{i \in \mathcal{I}} n_i \frac{\theta_i s_i^*(\mathbf{n})}{s_i^*(\mathbf{n}) + 1} \\ & \text{subject to} && n_i \in \{0, \dots, N_i\}, \quad i \in \mathcal{I} \end{aligned} \quad (2.10)$$

Let us first solve the CP_1 subproblem in \mathbf{s} . Note that it is a convex optimization problem. By using Lagrange multiplier technique, we can get the first-order necessary and sufficient condition:

$$s_i^*(\lambda) = \left(\sqrt{\frac{\theta_i}{\lambda}} - 1 \right)^+, \quad (2.11)$$

where λ is the Lagrange multiplier corresponding to the resource constraint (2.9).

Meanwhile, we note the resource constraint (2.9) must hold with equality, since the objective is a strictly increasing function in s_i . Thus, by plugging (2.11) into (2.9), we have

$$\sum_{i \in \mathcal{I}} n_i \left(\sqrt{\frac{\theta_i}{\lambda}} - 1 \right)^+ = S. \quad (2.12)$$

This weighted water-filling problem (where $\frac{1}{\sqrt{\lambda}}$ can be viewed as the water level) in general has no closed-form solution for λ . However, we can efficiently determine the optimal solution λ^* by exploiting the special structure of our problem. Note that since $\theta_1 > \dots > \theta_I$, then λ^* must satisfy the following condition:

$$\sum_{i=1}^{K^{cp}} n_i \left(\sqrt{\frac{\theta_i}{\lambda^*}} - 1 \right) = S, \quad (2.13)$$

for a group index threshold value K^{cp} satisfying

$$\frac{\theta_{K^{cp}}}{\lambda^*} > 1 \text{ and } \frac{\theta_{K^{cp}+1}}{\lambda^*} \leq 1. \quad (2.14)$$

In other words, only groups with index no larger than K_{cp} will be allocated the positive resource. This property leads to the following simple Algorithm 1 to compute λ^* and group index threshold K^{cp} : we start by assuming $K^{cp} = I$ and compute λ . If (2.14) is not satisfied, we decrease K^{cp} by one and recompute λ until (2.14) is satisfied.

Algorithm 1 Solving the Resource Allocation Problem CP_1

```

1: function  $CP(\{n_i, \theta_i\}_{i \in \mathcal{I}}, S)$ 
2:    $k \leftarrow I, \lambda(k) \leftarrow \left( \frac{\sum_{i=1}^k n_i \sqrt{\theta_i}}{S + \sum_{i=1}^k n_i} \right)^2$ 
3:   while  $\theta_k \leq \lambda(k)$  do
4:      $k \leftarrow k - 1, \lambda(k) \leftarrow \left( \frac{\sum_{i=1}^k n_i \sqrt{\theta_i}}{S + \sum_{i=1}^k n_i} \right)^2$ 
5:   end while
6:    $K^{cp} \leftarrow k, \lambda^* \leftarrow \lambda(k)$ 
7:   return  $K^{cp}, \lambda^*$ 
8: end function

```

Since $\theta_1 > \lambda(1) = \left(\frac{n_1}{s+n_1} \right)^2 \theta_1$, Algorithm 1 always converges and returns the unique values of K^{cp} and λ^* . The complexity is $\mathcal{O}(I)$, i.e., linear in the number of user groups (not the number of users).

With K^{cp} and λ^* , the solution of the resource allocation problem can be written as

$$s_i^* = \begin{cases} \sqrt{\frac{\theta_i}{\lambda^*}} - 1, & i = 1, \dots, K^{cp}; \\ 0, & \text{otherwise.} \end{cases} \quad (2.15)$$

For the ease of discussion, we introduce a new notion of the *effective market*, which denotes all the groups allocated non-zero resource. For the resource allocation subproblem CP_1 , the threshold K^{cp} describes the size of the effective market. All groups with indices no larger than K^{cp} are *effective groups*, and users in these groups as *effective users*. An example of the effective market is illustrated in Fig. 2.2.

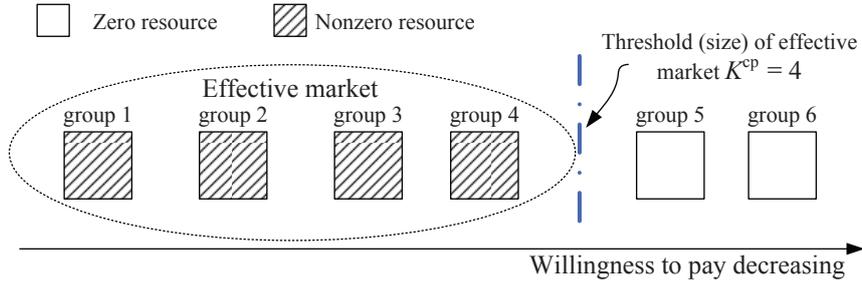


Figure 2.2: A 6-group example for effective market: the willingness to pay decreases from group 1 to group 6. The effective market threshold can be obtained by Algorithm 1, and is 4 in this example.

Now let us solve the admission control subproblem CP_2 . Denote the objective (2.10) as $R_{cp}(\mathbf{n})$, by (2.15), then $R_{cp}(\mathbf{n}) \triangleq \sum_{i=1}^{K^{cp}} n_i \left(\sqrt{\frac{\theta_i}{\lambda^*(\mathbf{n})}} - 1 \right) \sqrt{\theta_i \lambda^*(\mathbf{n})}$. We first relax the integer domain constraint of n_i as $n_i \in [0, N_i]$. Since (2.13), by taking the derivative of the objective function $R_{cp}(\mathbf{n})$ with respect to n_i , we have

$$\frac{\partial R_{cp}(\mathbf{n})}{\partial n_i} = n_i \left(\sqrt{\frac{\theta_i}{\lambda^*(\mathbf{n})}} - 1 \right) \frac{\partial \sqrt{\theta_i \lambda^*(\mathbf{n})}}{\partial n_i}, \quad (2.16)$$

Also from (2.13), we have $\lambda^* = \left(\frac{\sum_{i=1}^{K^{cp}} n_i \sqrt{\theta_i}}{S + \sum_{i=1}^{K^{cp}} n_i} \right)^2$, thus $\frac{\partial \sqrt{\lambda^*(\mathbf{n})}}{\partial n_i} > 0$, for $i = 1, \dots, K^{cp}$, and $\frac{\partial \sqrt{\lambda^*(\mathbf{n})}}{\partial n_i} = 0$, for $i = K^{cp} + 1, \dots, I$. This means that the objective $R_{cp}(\mathbf{n})$ is strictly increasing in n_i for all $i = 1, \dots, K^{cp}$, thus it is optimal to admit all users in the effective market. The admission decisions for the groups not in the

effective market is irrelevant to the optimization, since those users consume zero resource. Therefore, one of the optimal solutions of CP_1 subproblem is $n_i^* = N_i$ for all $i \in \mathcal{I}$. Solving CP_1 and CP_2 subproblems leads to the optimal solution of the CP problem:

Theorem 2.1. *There exists an optimal solution of the CP problem that satisfies the following conditions:*

- *All users are admitted: $n_i^* = N_i$ for all $i \in \mathcal{I}$.*
- *There exist a value λ^* and a group index threshold $K^{cp} \leq I$, such that only the top K^{cp} groups of users receive positive resource allocations,*

$$s_i^* = \begin{cases} \sqrt{\frac{\theta_i}{\lambda^*}} - 1, & i = 1, \dots, K^{cp}; \\ 0, & \text{otherwise.} \end{cases}$$

with the prices

$$p_i^* = \begin{cases} \sqrt{\theta_i \lambda^*}, & i = 1, \dots, K^{cp}; \\ \theta_i, & \text{otherwise.} \end{cases}$$

The values of λ^ and K^{cp} can be computed as in Algorithm 1 by setting $n_i = N_i$, for all $i \in \mathcal{I}$.*

Theorem 2.1 provides the right economic intuition: service provider maximizes its revenue by charging a higher price to users with a higher willingness to pay. It is easy to check that $p_i > p_j$ for any $i < j$. The users with low willingness to pay are excluded from the markets.

2.3.3 Properties

Here we summarize some interesting properties of the optimal complete price differentiation scheme:

2.3.3.1 Threshold structure

The threshold based resource allocation means that groups with higher willingness to pay have higher priorities of obtaining the resource at the optimal solution.

To see this clearly, assume the effective market size is $K^{(1)}$ under parameters $\{\theta_i, N_i^{(1)}\}_{i \in \mathcal{I}}$ and S . Here the superscript (1) denotes the first parameter set. Now consider another set of parameters $\{\theta_i, N_i^{(2)}\}_{i \in \mathcal{I}}$ and S , where $N_i^{(2)} \geq N_i^{(1)}$ for each group i and the new effective market size is $K^{(2)}$. By (2.13), we can see that $K^{(2)} \leq K^{(1)}$. Furthermore, we can show that if some high willingness to pay group has many more users under the latter system parameters, i.e., $N_i^{(2)}$ is much larger than $N_i^{(1)}$ for some $i < K^{(1)}$, then the effective size will be strictly decreased, i.e., $K^{(2)} < K^{(1)}$. In other words, the increase of users with high willingness to pay will drive the users with low willingness to pay out of the effective market.

2.3.3.2 Admission control with pricing

Theorem 2.1 shows the explicit admission control is not necessary at the optimal solution. Also from Theorem 2.1, we can see that when the number of users in any effective group increases, the price p_i^* , for all $i \in \mathcal{I}$ increases and resource s_i^* , for all $\forall i \leq K^{cp}$ decreases. The prices serve as the indications of the scarcity of the resources and will automatically prevent the users with low willingness to pay to consume the network resource.

2.4 Single Pricing Scheme

In last section, we showed that the CP scheme is the optimal pricing scheme to maximize the revenue under complete information. However, such a complicated

pricing scheme is of high implementational complexity. In this section we study the single pricing scheme. It is clear that the scheme will in general suffer a revenue loss comparing with the CP scheme. We will try to characterize the impact of various system parameters on such revenue loss.

2.4.1 Problem Formulation and Solution

Let us first formulate the Single Pricing (SP) problem.

$$\begin{aligned}
 SP : \quad & \underset{p \geq 0, \mathbf{n}}{\text{maximize}} && p \sum_{i \in \mathcal{I}} n_i s_i \\
 & \text{subject to} && s_i = \left(\frac{\theta_i}{p} - 1 \right)^+, \quad i \in \mathcal{I} \\
 & && n_i \in \{0, \dots, N_i\}, \quad i \in \mathcal{I} \\
 & && \sum_{i \in \mathcal{I}} n_i s_i \leq S.
 \end{aligned}$$

Comparing with the CP problem in Section 2.3, here the service provider charges a single price p to all groups of users. After a similar transformation as in Section 2.3, we can show that the optimal single price satisfies the following the weighted water-filling condition

$$\sum_{i \in \mathcal{I}} N_i \left(\frac{\theta_i}{p} - 1 \right)^+ = S.$$

Thus we can obtain the following solution that shares a similar structure as complete price differentiation.

Theorem 2.2. *There exists an optimal solution of the SP problem that satisfies the following conditions:*

- *All users are admitted: $n_i^* = N_i$, for all $i \in \mathcal{I}$.*

- There exist a price p^* and a group index threshold $K^{sp} \leq I$, such that only the top K^{sp} groups of users receive positive resource allocations,

$$s_i^* = \begin{cases} \frac{\theta_i}{p^*} - 1, & i = 1, 2, \dots, K^{sp}, \\ 0, & \text{otherwise,} \end{cases}$$

with the price

$$p^* = p(K^{sp}) = \frac{\sum_{i=1}^{K^{sp}} N_i \theta_i}{S + \sum_{i=1}^{K^{sp}} N_i}.$$

The value of K^{sp} and p^* can be computed as in Algorithm 2.

Algorithm 2 Search the threshold of the SP problem

```

1: function  $SP(\{N_i, \theta_i\}_{i \in \mathcal{I}}, S)$ 
2:    $k \leftarrow I, p(k) \leftarrow \frac{\sum_{i=1}^k N_i \theta_i}{S + \sum_{i=1}^k N_i}$ 
3:   while  $\theta_k \leq p(k)$  do
4:      $k \leftarrow k - 1, p(k) \leftarrow \frac{\sum_{i=1}^k N_i \theta_i}{S + \sum_{i=1}^k N_i}$ 
5:   end while
6:    $K^{sp} \leftarrow k, p^* \leftarrow p(k)$ 
7:   return  $K^{sp}, p^*$ 
8: end function

```

2.4.2 Properties

The SP scheme shares several similar properties as the CP scheme (Sec. 2.3.3), including the threshold structure and admission control with pricing. Similarly, we can define the effective market for the SP scheme.

It is more interesting to notice the differences between these two schemes. To distinguish solutions, we use the superscript “cp” for the CP scheme, and “sp” for the SP scheme.

Proposition 2.1. Under same parameters $\{N_i, \theta_i\}_{i \in \mathcal{I}}$ and S :

1. The effective market of the *SP* scheme is no larger than the one of the *CP* scheme, i.e., $K^{sp} \leq K^{cp}$.
2. There exists a threshold $\bar{k} \in \{1, 2, \dots, K^{sp}\}$, such that
 - Groups with indices less than \bar{k} (users with high willingness to pay) are charged with higher prices and allocated less resources in the *CP* scheme, i.e., $p_i^{cd} \geq p^*$ and $s_i^{cd} \leq s_i^{sp}$, $\forall i \leq \bar{k}$, where the equality holds if only if $i = \bar{k}$ and $\theta_{\bar{k}} = \frac{p^*{}^2}{\lambda^*}$,
 - Groups with indices greater than \bar{k} (users with low willingness to pay) are charged with lower prices and allocated more resources in the *CP* scheme, i.e., $p_i^{cd} < p^*$ and $s_i^{cd} > s_i^{sp}$, $\forall i \geq \bar{k}$.

where p^* is the optimal single price.

The proof is given in Appendix 2.10.2. An illustrative example is shown in Fig. 2.3 and Fig. 2.4.

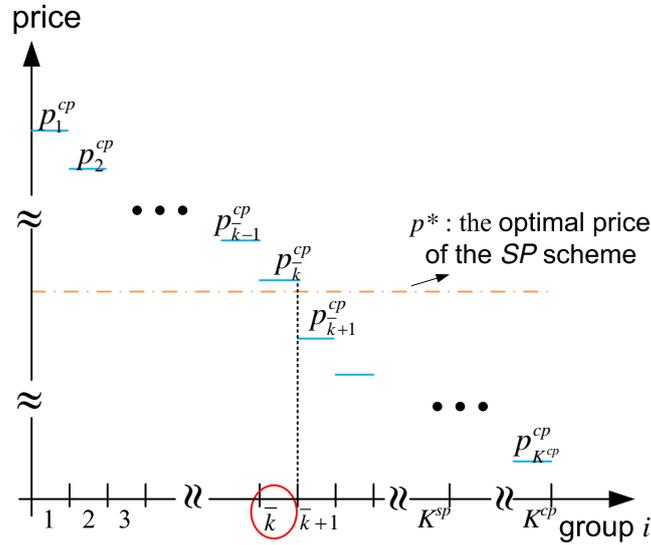


Figure 2.3: Comparison of prices between the *CP* scheme and the *SP* scheme

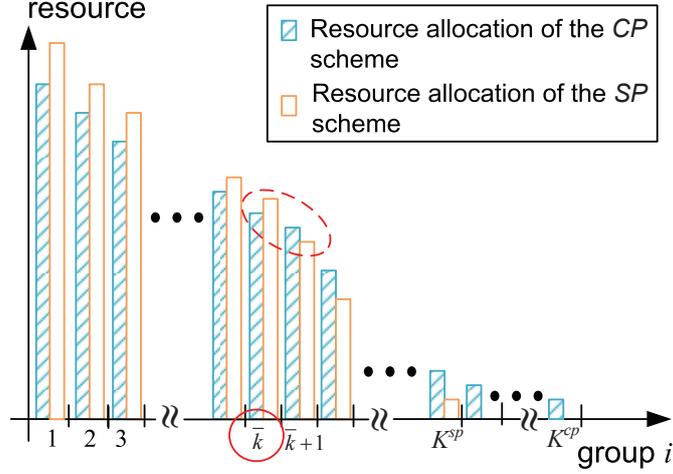


Figure 2.4: Comparison of resource allocation between the CP scheme and the SP scheme

It is easy to understand that the SP scheme makes less revenue, since it is a feasible solution to the CP problem. A little bit more computation sheds more light on this comparison. We introduce the following notations to streamline the comparison:

- $N_{eff}(k) \triangleq \sum_{i=1}^k N_i$: the number of effective users, where k is the size of the effective market.
- $\gamma_i(k) \triangleq \frac{N_i}{N_{eff}(k)}$, $i = 1, 2, \dots, k$: the fraction of group i 's users in the effective market.
- $\bar{s}(k) \triangleq \frac{S}{N_{eff}(k)}$: the average resource per an effective user.
- $\bar{\theta}(k) \triangleq \sum_{i=1}^k \gamma_i \theta_i$: the average willingness to pay per an effective user.

Based on Theorem 2.1, the revenue of the CP scheme is

$$R^{cp}(K^{cp}) = N_{eff}(K^{cp}) \left(\frac{\bar{s}(K^{cp}) \bar{\theta}(K^{cp}) + g(K^{cp})}{\bar{s}(K^{cp}) + 1} \right), \quad (2.17)$$

where

$$g(K^{cp}) = \frac{1}{\lambda^*} \sum_{i=1}^{K^{cp}} \sum_{j>i}^{K^{cp}} \gamma_i \gamma_j (p_i^{cp} - p_j^{cp})^2. \quad (2.18)$$

Based on Theorem 2.2, the revenue of the SP scheme is

$$R^{sp}(K^{sp}) = N_{eff}(K^{sp}) \left(\frac{\bar{s}(K^{sp})\bar{\theta}(K^{sp})}{\bar{s}(K^{sp}) + 1} \right). \quad (2.19)$$

From (2.17) and (2.19), it is clear to see that $R^{cp} \geq R^{sp}$ due to two factors: one is the non-negative term in (2.18), the other is $K^{cp} \geq K^{sp}$: a higher level of differentiation implies a no smaller effective market. Let us further discuss them in the following two cases:

- If $K^{cp} = K^{sp}$, then the additional term of (2.18) in (2.17) means that $R^{cp} \geq R^{sp}$. The equality holds if and only if $K^{cp} = 1$, in which case $g(K^{cp}) = 0$. Note that in this case, the CP scheme degenerates to the SP scheme. We name the nonnegative term $g(K^{cp})$ in (2.18) as *price differentiation gain*, as it measures the average price difference between any effective users in the CP scheme. The larger the price difference, the larger the gain. When there is no differentiation in the degenerating case ($K^{cp} = 1$), the gain is zero.
- If $K^{cp} > K^{sp}$, since the common part of two revenue $N_{eff}(K) \left(\frac{\bar{s}(K)\bar{\theta}(K)}{\bar{s}(K)+1} \right) = \frac{S\bar{\theta}N_{eff}(K)}{S+N_{eff}(K)}$ is a strictly increasing function of $N_{eff}(K)$, price differentiation makes more revenue even if the positive differentiation gain $g(K^{cp})$ is not taken into consideration. This result is intuitive, that more consumers with purchasing power always mean more revenue in the service provider's pocket.

Finally, we note that the CP scheme in Section 2.3 requires the complete network information. The SP scheme here, on the other hand, works in the incomplete information case as well. This distinction becomes important in Section 2.6.

2.5 Partial Price Differentiation under Complete Information

For a service provider facing thousands of user types, it is often impractical to design a price choice for each user type. The reasons behind this, as discussed in [26], are mainly high system overheads and customers' aversion. However, as we have shown in Sec. 2.4, the single pricing scheme may suffer a considerable revenue loss. How to achieve a good tradeoff between the implementational complexity and the total revenue? In reality, we usually see that the service provider offers only a few pricing plans for the entire users population; we term it as the *partial price differentiation* scheme. In this section, we will answer the following question: if the service provider is constrained to maintain a limited number of prices, p^1, \dots, p^J , $J \leq I$, then what is the optimal pricing strategy and the maximum revenue? Concretely, the Partial Price differentiation (*PP*) problem is formulated as follows.

$$PP : \begin{aligned} & \underset{n_i, p_i, s_i, p^j, a_i^j}{\text{maximize}} && \sum_{i \in \mathcal{I}} n_i p_i s_i \\ & \text{subject to} && s_i = \left(\frac{\theta_i}{p_i} - 1 \right)^+, \forall i \in \mathcal{I}, \end{aligned} \quad (2.20)$$

$$n_i \in \{0, \dots, N_i\}, \forall i \in \mathcal{I}, \quad (2.21)$$

$$\sum_{i \in \mathcal{I}} n_i s_i \leq S, \quad (2.22)$$

$$p_i = \sum_{j \in \mathcal{J}} a_i^j p^j, \quad (2.23)$$

$$\sum_{j \in \mathcal{J}} a_i^j = 1, a_i^j \in \{0, 1\}, \forall i \in \mathcal{I}. \quad (2.24)$$

Here \mathcal{J} denotes the set $\{1, 2, \dots, J\}$. Since we consider the complete information scenario in this section, the service provider can choose the price charged

to each group, thus constraints (2.20) – (2.22) are the same as in the CP problem. Constraints (2.23) and (2.24) mean that p_i charged to each group i is one of J choices from the set $\{p^j, j \in \mathcal{J}\}$. For convenience, we define *cluster* $\mathcal{C}^j \triangleq \{i \mid a_i^j = 1\}$, $j \in \mathcal{J}$, which is a set of groups charged with the same price p^j . We use superscript j to denote clusters, and subscript i to denote groups through this section. We term the binary variables $\mathbf{a} \triangleq \{a_i^j, j \in \mathcal{J}, i \in \mathcal{I}\}$ as the *partition*, which determines which cluster each group belongs to.

The PP problem is a combinatorial optimization problem, and is more difficult than the previous CP and SP problems. On the other hand, we notice that this PP problem formulation includes the CP scheme ($J = I$) and the SP scheme scenario ($J = 1$) as special cases. The insights we obtained from solving these two special cases in Sections 2.3 and 2.4 will be helpful to solve the general PP problem.

2.5.1 Three-level Decomposition

To solve the PP problem, we decompose and tackle it in three levels. In the lowest level-3, we determine the pricing and resource allocation for each cluster, given a fixed partition and fixed resource allocation among clusters. In level-2, we compute the optimal resource allocation among clusters, given a fixed partition. In level-1, we optimize the partition among groups.

2.5.1.1 Level-3: Pricing and Resource Allocation in Each Cluster

For a fix partition \mathbf{a} and a cluster resource allocation $\mathbf{s} \triangleq \{s^j\}_{j \in \mathcal{J}}$, we focus the pricing and resource allocation problems within each cluster \mathcal{C}^j , $j \in \mathcal{J}$:

$$\begin{aligned} \text{Level-3: maximize}_{n_i, s_i, p^j} \quad & \sum_{i \in \mathcal{C}^j} n_i p^j s_i \\ \text{subject to} \quad & s_i = \left(\frac{\theta_i}{p^j} - 1 \right)^+, \quad \forall i \in \mathcal{C}^j, \\ & n_i \leq N_i, \quad \forall i \in \mathcal{C}^j, \\ & \sum_{i \in \mathcal{C}^j} n_i s_i \leq s^j. \end{aligned}$$

The level-3 subproblem coincides with the *SP* scheme discussed in Section 2.4, since all groups within the same cluster \mathcal{C}^j are charged with a single price p^j . We can then directly apply the results in Theorem 2 to solve the level-3 subproblem. We denote the effective market threshold² for cluster \mathcal{C}^j as K^j , which can be computed in Algorithm 2. An illustrative example is shown in Fig. 2.5, where the cluster contains four groups (group 4, 5, 6 and 7), and the effective market contains groups 4 and 5, thus $K^j = 5$. The service provider obtains the following maximum revenue obtained from cluster \mathcal{C}^j :

$$R^j(s^j, \mathbf{a}) = \frac{s^j \sum_{i \in \mathcal{C}^j, i \leq K^j} N_i \theta_i}{s^j + \sum_{i \in \mathcal{C}^j, i \leq K^j} N_i}. \quad (2.25)$$

²Note that we do not assume that the effective market threshold equals to the number of effective groups, e.g., there are 2 effective groups in Fig. 5, but threshold $K^j = 5$. Later we will prove that there is unified threshold for the *PP* problem. Then by this result, the group index threshold actually coincides with the number of effective groups.

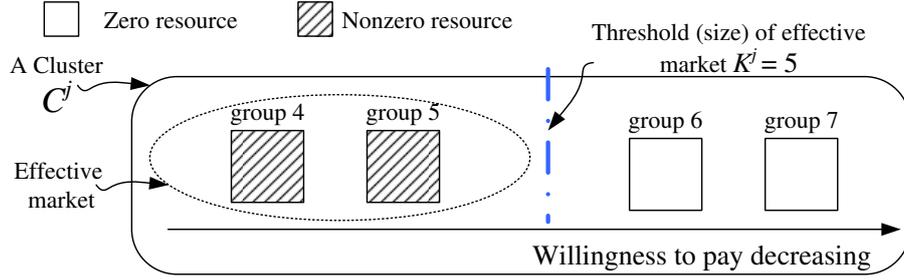


Figure 2.5: An illustrative example: the cluster contains four groups, group 4, 5, 6 and 7; and the effective market contains group 4 and 5, thus $K^j = 5$

2.5.1.2 Level-2: Resource Allocation among Clusters

For a fix partition \mathbf{a} , we then consider the resource allocation among clusters.

$$\begin{aligned} \text{Level-2: } & \underset{s^j \geq 0}{\text{maximize}} && \sum_{j \in \mathcal{J}} R^j(s^j, \mathbf{a}) \\ & \text{subject to} && \sum_{j \in \mathcal{J}} s^j \leq S \end{aligned}$$

We will show in Section 2.5.2 that subproblems in level-2 and level-3 can be transformed into a complete price differentiation problem under proper technique conditions. Let us denote the its optimal value as $R_{pp}(\mathbf{a})$.

2.5.1.3 Level-1: cluster partition

Finally, we solve the cluster partition problem.

$$\begin{aligned} \text{Level-1: } & \underset{a_i^j \in \{0,1\}}{\text{maximize}} && R_{pp}(\mathbf{a}) \\ & \text{subject to} && \sum_{j \in \mathcal{J}} a_i^j = 1, \quad i \in \mathcal{I}. \end{aligned}$$

This partition problem is a combinatorial optimization problem. The size of its feasible set is $S(I, J) = \frac{1}{J!} \sum_{t=1}^J (-1)^{J+t} C(J, t) t^I$, *Stirling number of the second kind* [41, Chap.13], where $C(J, t)$ is the binomial coefficient. Some numerical

Table 2.1: Numerical examples for feasible set size of the partition problem in Level-1

Number of groups	$I = 10$		$I = 100$		$I = 1000$
	$J = 2$	$J = 3$	$J = 2$	$J = 3$	$J = 2$
$S(I, J)$	511	9330	6.338×10^{29}	8.590×10^{46}	5.358×10^{300}
$C(I - 1, J - 1)$	9	36	99	4851	999

examples are given in the third row in Table 2.1. If the number of prices J is given, the feasible set size is exponential in the total number of groups I . For our problem, however, it is possible to reduce the size of the feasible set by exploiting the special problem structure. More specifically, the group indices in each cluster should be consecutive at the optimum. This means that the size of the feasible set is $C(I - 1, J - 1)$ as shown in the last row in Table 2.1, and thus is much smaller than $S(I, J)$.

Next we discuss how to solve the three level subproblems. A route map for the whole solution process is given in Fig. 2.6.

2.5.2 Solving Level-2 and Level-3

The optimal solution (2.25) of the level-3 problem can be equivalently written as

$$R^j(\mathbf{s}, \mathbf{a}) = \frac{s^j \sum_{i \in C^j, i \leq K^j} N_i \theta_i}{s^j + \sum_{i \in C^j, i \leq K^j} N_i} \stackrel{(a)}{=} \frac{s^j N^j \theta^j}{s^j + N^j}, \quad (2.26)$$

$$\text{where } \begin{cases} N^j &= \sum_{i \in C^j, i \leq K^j} N_i, \\ \theta^j &= \sum_{i \in C^j, i \leq K^j} \frac{N_i \theta_i}{N^j}. \end{cases} \quad (2.27)$$

The equality (a) in (2.26) means that each cluster C^j can be equivalently treated as a group with N^j homogeneous users with the same willings to pay θ^j . We name

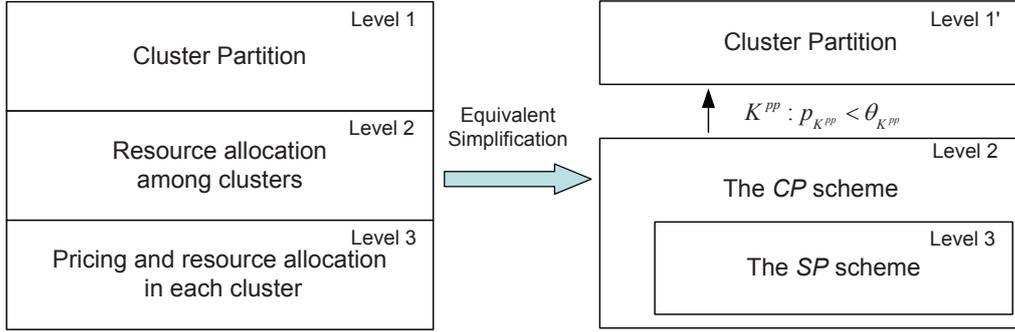


Figure 2.6: Decomposition and simplification of the general PP problem: The three-level decomposition structure of the PP problem is shown in the left hand side. After simplifications in Section 2.5.2 and 2.5.3, the problem will be reduced to structure in right hand side.

this equivalent group as a *super-group* (SG). We summarize the above result as the following lemma.

Lemma 2.1. *For every cluster C^j and total resource s^j , $j \in \mathcal{J}$, we can find an equivalent super-group which satisfies conditions in (2.27) and achieves the same revenue under the SP scheme.*

Based on Lemma 2.1, level-2 and level-3 subproblems together can be viewed as the CP problem for super-groups. Since a cluster and its super-group from a one-to-one mapping, we will use the two words interchangeably in the sequel.

However, simply combining Theorems 2.1 and 2.2 to solve level-2 and level-3 subproblems for a fixed partition α can result in a very high complexity. This is because the effective markets within each super-group and between super-groups are coupled together. An illustrative example of this coupling effective market is shown in Fig. 2.7, where K^c is the threshold between clusters and has three possible positions (i.e., between group 2 and group 3, between group 5 and group 6, or after group 6); and K_1 and K_2 are thresholds within cluster \mathcal{C}^1 and \mathcal{C}^2 , which

have two or three possible positions, respectively. Thus, there are $(2 \times 3) \times 3 = 18$ possible thresholds possibilities in total.

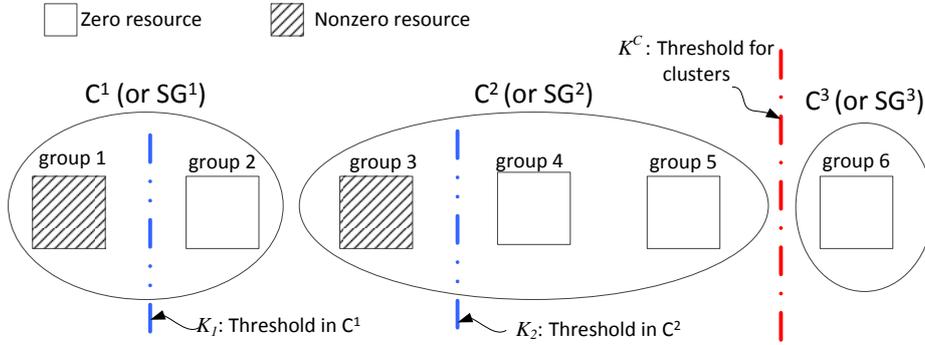


Figure 2.7: An example of coupling thresholds.

The key idea resolving this coupling issue is to show that the situation in Fig. 2.7 can not be an optimal solution of the PP problem. The results in Sections 2.3 and 2.4 show that there is a unified threshold at the optimum in both the CP and SP cases, e.g., Fig. 2.2. Next we will show that a unified single threshold also exists in the PP case.

Lemma 2.2. *At any optimal solution of the PP scheme, the group indices of the effective market is consecutive.*

The proof of Lemma 2.2 can be found in Appendix 2.10.3. The intuition is that the resource should be always allocated to users with high willingness to pay at the optimum. Thus, it is not possible to have Fig. 2.7 at an optimal solution, where users in group 2 with high willingness to pay are allocated zero resource while users in group 3 with low willingness to pay are allocated positive resources.

Based on Lemma 2.2, we know that there is a unified effective market threshold for the PP problem, denoted as K^{pp} . Since all groups with indices larger than K^{pp} make zero contribution to the revenue, we can ignore them and only

consider the partition problem for the first K^{pp} groups. Given a partition that divides the K^{pp} groups into J clusters (super-groups), we can apply the CP result in Section 2.3 to compute the optimal revenue in the level-2 subproblem based on Theorem 2.1.

$$\begin{aligned} R_{pp}(\mathbf{a}) &= \sum_{j=1}^J N^j \theta^j - \frac{\left(\sum_{j=1}^J N^j \sqrt{\theta^j}\right)^2}{S + \sum_{j=1}^J N^j} \\ &= \sum_{i=1}^{K^{pp}} N_i \theta_i - \frac{\left(\sum_{j=1}^J N^j \sqrt{\theta^j}\right)^2}{S + \sum_{i=1}^{K^{pp}} N_i}. \end{aligned} \quad (2.28)$$

2.5.3 Solving Level-1

2.5.3.1 With a given effective market threshold K^{pp}

Based on the previous results, we first simplify the level-1 subproblem, and prove the theorem below.

Theorem 2.3. *For a given threshold K^{pp} , the optimal partition of the level-1 subproblem is the solution of the following optimization problem.*

$$\begin{aligned} \text{Level-1'} \quad & \underset{\alpha_i^j, N^j, \theta^j}{\text{minimize}} \quad \sum_{j \in \mathcal{J}} N^j \sqrt{\theta^j} \\ & \text{subject to} \quad N^j = \sum_{i \in \mathcal{K}^{pp}} N_i \alpha_i^j, \quad j \in \mathcal{J}, \\ & \quad \quad \quad \theta^j = \sum_{i \in \mathcal{K}^{pp}} \frac{N_i \alpha_i^j}{N^j} \theta_i, \quad j \in \mathcal{J}, \\ & \quad \quad \quad \sum_{j \in \mathcal{J}} \alpha_i^j = 1, \quad \alpha_i^j \in \{0, 1\}, \quad i \in \mathcal{K}^{pp}, \quad j \in \mathcal{J}, \\ & \quad \quad \quad \theta_{K^{pp}} > p^J = \sqrt{\theta^J(\mathbf{a}) \lambda(\mathbf{a})}. \end{aligned} \quad (2.29)$$

where $\mathcal{K}^{pp} \triangleq \{1, 2, \dots, K^{pp}\}$, $\theta^J(\mathbf{a})$ is the value of average willingness to pay of the J th group for the partition \mathbf{a} , and $\lambda(\mathbf{a}) = \left(\frac{\sum_{j \in \mathcal{J}} N^j \sqrt{\theta^j}}{S + \sum_{i=1}^{K^{pp}} N_i}\right)^2$.

Proof. The objective function and the first three constraints in the level-1 subproblem are easy to understand: if the effective market threshold K^{pp} is given, then the objective function of the level-1 subproblem, maximizing R_{pp} in (2.28) over \mathbf{a} , is as simple as minimizing $\sum_{j=1}^J N^j \sqrt{\theta^j}$ as the level-1 subproblem suggested; the first three constraints are given by the definition of the partition.

Constraint (2.29) is the threshold condition that supports (2.28), which means that group with the least willingness to pay in the effective market has a positive demand. It ensures that calculating the revenue by (2.28) is valid. Remember that the solution of the *CP* problem of level-2 and level-3 is threshold based, and Lemma 2.2 indicates that (2.29) is sufficient for that all groups with willingness to pay larger than group K^{pp} can have positive demands. Otherwise, we can construct another partition leading to a larger revenue (please refer to the proof of Lemma 2.2), or equivalently leading to a less objective value of the level-1 subproblem. This leads to a contradiction. \square

The level-1 subproblem is still a combinatorial optimization problem with a large feasible set of \mathbf{a} (similar as the original level-1 subproblem). The following result can help us to reduce the size of the feasible set.

Theorem 2.4. *For any effective market size K^{pp} and number of prices J , an optimal partition of the *PP* problem involves consecutive group indices within clusters.*

The proof of Theorem 2.4 is given in Appendix 2.10.4. We first prove this result is true for the level-1 subproblem without constraint (2.29), and further show that this result will not be affected by (2.29). The intuition is that users with high willingness to pay should be allocated positive resources with priority. It implies that groups with similar willingness to pay should be partitioned in the same cluster, instead of in several far away clusters. Or equivalently, the group indices

within each cluster should be consecutive.

We define \mathcal{A} as the set of all partitions with consecutive group indices within each cluster, and $v(\mathbf{a}) = \sum_{j \in \mathcal{J}} N^j \sqrt{\theta^j}$ is the value of objective of the level-1 subproblem for a partition \mathbf{a} . Algorithm 3 finds the optimal solution of the level-1 subproblem. The main idea for this algorithm is to enumerate every possible partition in set \mathcal{A} , and then check whether the threshold condition (2.29) can be satisfied. The main part of this algorithm is to enumerate all partitions in set \mathcal{A} of $\mathcal{C}(K^{pp} - 1, J - 1)$ feasible partitions. Thus the complexity of Algorithm 3 is no more than $\mathcal{O}((K^{pp})^{J-1})$.

Algorithm 3 Solve the level-1 subproblem with fixed K^{pp}

```

1: function LEVEL-1( $K^{pp}, J$ )
2:    $k \leftarrow K^{pp}$ 
3:    $v^* \leftarrow \sqrt{\sum_{i=1}^k N_i \sum_{i=1}^k N_i \theta_i}$ ,  $\mathbf{a}^* = \mathbf{0}$ 
4:   for  $\mathbf{a} \in \mathcal{A}$  do
5:     if  $\theta_k > \sqrt{\theta^J(\mathbf{a}) \lambda(\mathbf{a})}$  then
6:       if  $v(\mathbf{a}) < v^*$  then
7:          $v^* \leftarrow v(\mathbf{a})$ ,  $\mathbf{a}^* \leftarrow \mathbf{a}$ 
8:       end if
9:     end if
10:  end for
11:  return  $\mathbf{a}^*$ 
12: end function

```

2.5.3.2 Search the optimal effective market threshold K^{pp}

We know the optimal market threshold K^{pp} is upper-bounded, i.e., $K^{pp} \leq K^{cp} \leq I$. Thus we can first run Algorithm 1 to calculate the effective market size for the

CP scheme K^{cp} . Then, we search the optimal K^{pp} iteratively using Algorithm 3 as an inner loop. We start by letting $K^{pp} = K^{cp}$ and run Algorithm 3. If there is no solution, we decrease K^{pp} by one and run Algorithm 3 again. The algorithm will terminate once we find an effective market threshold where Algorithm 3 has an optimal solution. Once the optimal threshold and the partition of the clusters are determined, we can further run Algorithm 1 to solve the joint optimal resource allocation and pricing scheme. The pseudo code is given in Algorithm 4 as follows.

Algorithm 4 Solve partial price differentiation problem

```

1:  $p_i \leftarrow \theta_i$ 
2:  $k \leftarrow \text{CP}(\{N_i, \theta_i\}_{i \in \mathcal{I}}, S)$ -1,  $\mathbf{a}^* \leftarrow \text{Level-1}(k, J)$ 
3: while  $\mathbf{a}^* \neq \mathbf{0}$  do
4:    $k \leftarrow k - 1$ ,  $\mathbf{a}^* \leftarrow \text{Level-1}(k, J)$ 
5: end while
6: for  $j \leftarrow 1, J$  do
7:    $N^j \leftarrow \sum_{i=1}^k N_i a_i^j$ ,  $\theta^j \leftarrow \sum_{i=1}^k \frac{N_i a_i^j}{N^j} \theta_i$ 
8: end for
9:  $\lambda \leftarrow \text{CP}(\{N^j, \theta^j\}_{i \in \mathcal{J}}, S)$ -2
10: for  $i \leftarrow 1, k$  do
11:    $p_i \leftarrow \sum_{j=1}^J a_i^j \sqrt{\theta^j \lambda}$ 
12: end for
13: return  $\{p_i\}_{i \in \mathcal{I}}$ 

```

In Algorithm 4, it invokes two functions: $\text{CP}(\{N_i \theta_i\}_{i \in \mathcal{I}}, S)$ as described in Algorithm 1 and $\text{Level-1}(k, J)$ as in Algorithm 3. $\text{CP}(\{N_i \theta_i\}_{i \in \mathcal{I}}, S)$ returns a vector with two elements: $\text{CP}(\{N_i \theta_i\}_{i \in \mathcal{I}}, S)$ -1 denotes the first element K^{cp} , and $\text{CP}(\{N_i \theta_i\}_{i \in \mathcal{I}}, S)$ -2 denotes the second element λ^* in the CP problem.

The above analysis leads to the following theorem:

Theorem 2.5. *The solution obtained by Algorithm 4 is optimal for the PP problem.*

Proof. It is clear that Algorithm 4 enumerates every possible value of the effective market size for the PP problem K^{pp} , and for a given K^{pp} its inner loop Algorithm 3 enumerates every possible partition in set \mathcal{A} . Therefore, the result in Theorem 4 follows. \square

Next we discuss the complexity of Algorithm 4. The complexity of Algorithm 1 is $\mathcal{O}(I)$, and we run it twice in Algorithm 4. The worst case complexity of Algorithm 3 is $\mathcal{O}(I^{J-1})$, and we run it no more than $I - J$ times. Thus the whole complexity of Algorithm 4 is no more than $\mathcal{O}(I^J)$, which is polynomial of I .

2.6 Price Differentiation under Incomplete Information

In Sections 2.3, 2.4, and 2.5, we discuss various pricing schemes with different implementational complexity level under complete information, the revenues of which can be viewed as the benchmark of practical pricing designs. In this section, we further study the incomplete information scenario, where the service provider does not know the group association of each user. The challenge for pricing in this case is that the service provider needs to provide the right incentive so that a group i user does not want to pretend to be a user in a different group. It is clear that the CP scheme in Section 2.3 and the PP scheme in Section 2.5 cannot be directly applied here. The SP scheme in Section 2.4 is a special case, since it does not require the user-group association information in the first place and thus

can be applied in the incomplete information scenario directly. On the other hand, we know that the SP scheme may suffer a considerable revenue loss compared with the CP scheme. Thus it is natural to ask whether it is possible to design an incentive compatible differentiation scheme under incomplete information. In this section, we design a quantity-based price menu to incentivize the users to make the right self-selection and achieve the same maximum revenue of the CP scheme under complete information under proper technical conditions. We name it the Incentive Compatible Complete Price differentiation ($ICCP$) scheme.

In the $ICCP$ scheme, the service provider publishes the quantity-based price menu, which consists of several step functions of resource quantities. Users are allowed to freely choose their quantities. The aim of this price menu is to make the users *self-differentiated*, so that to mimic the same result (the same prices and resource allocations) of the CP scheme under complete information. Based on Theorem 2.1, there are only K (without confusion, we remove the superscript “cp” to simplify the notation) effective groups of users receiving non-zero resource allocations, thus there are K steps of unit prices, $p_1^* > p_2^* > \dots > p_K^*$ in the price menu. These prices are exactly the same optimal prices that the service provider would charge for K effective groups as in Theorem 2.1. Note that for the $K + 1, \dots, I$ groups, all the prices in the menu are too high for them, then they will still demand zero resource. The quantity is divided into K intervals by $K - 1$ thresholds, $s_{th}^1 > s_{th}^2 > \dots > s_{th}^{K-1}$. The $ICCP$ scheme can be specified as follows:

$$p(s) = \begin{cases} p_1^* & \text{when } s > s_{th}^1 \\ p_2^* & \text{when } s_{th}^1 \geq s > s_{th}^2 \\ \vdots & \\ p_K^* & \text{when } s_{th}^{K-1} \geq s > 0. \end{cases} \quad (2.30)$$

A four-group example is shown in Fig. 2.8.

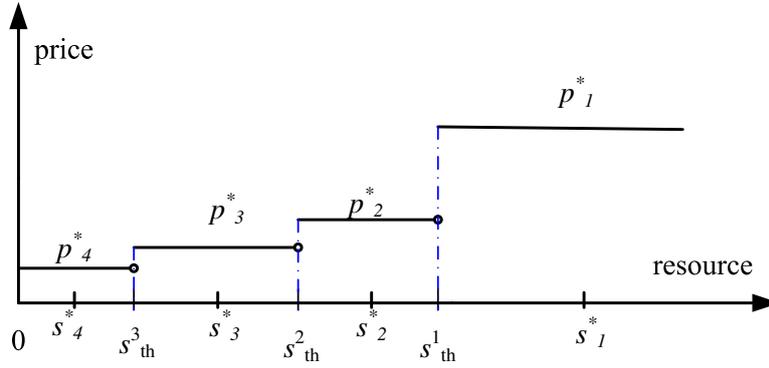


Figure 2.8: A four-group example of the *ICCP* scheme: where the prices $p_1^* > p_2^* > p_3^* > p_4^*$ are the same as the *CP* scheme. To mimic the same resource allocation as under the *CP* scheme, one necessary (but not sufficient) condition is $s_{th}^{j-1} \geq s_j^*$ for all j , where s_j^* is the optimal resource allocation of the *CP* scheme.

Note that in contrast to the usual “volume discount”, here the price is non-decreasing in quantity. This is motivated by the resource allocation in Theorem 2.1, that a user with a higher θ_i is charged a higher price for a larger resource allocation. Thus the observable quantity can be viewed as an indication of the unobservable users’ willingness to pay, and help to realize price differentiation under incomplete information.

The key challenge in the *ICCP* scheme is to properly set the quantity thresholds so that users are perfectly segmented through self-differentiation. This is, however, not always possible. Next we derive the necessary and sufficient conditions to guarantee the perfect segmentation.

Let us first study the self-selection problem between two groups: group i and group q with $i < q$. Later on we will generalize the results to multiple groups. Here group i has a higher willingness to pay, but will be charged with a higher price p_i^* in the *CP* case. The incentive compatible constraint is that a user with

high willingness to pay can not get more surplus by pretending to be a user with low willingness to pay, i.e., $\max_s U_i(s; p_i^*) \geq \max_s U_i(s; p_q^*)$, where $U_i(s; p) = \theta_i \ln(1+s) - ps$ is the surplus of a group i user when it is charged with price p .

Without confusion, we still use s_i^* to denote the optimal resource allocation under the optimal prices in Theorem 2.1, i.e., $s_i^* = \arg \max_{s_i \geq 0} U_i(s_i; p_i^*)$. We define $s_{i \rightarrow q}$ as the quantity satisfying

$$\begin{cases} U_i(s_{i \rightarrow q}; p_q^*) = U_i(s_i^*; p_i^*) \\ s_{i \rightarrow q} < s_i^* \end{cases} . \quad (2.31)$$

In other words, when a group i user is charged with a lower price p_q^* and demands resource quantity at $s_{i \rightarrow q}$, it achieves the same as the maximum surplus under the optimal price of the CP scheme p_i^* , as showed in Fig. 2.9. Since there are two solutions of the first equation of (2.31), we constraint $s_{i \rightarrow q}$ to be the one that is smaller than s_i^* .

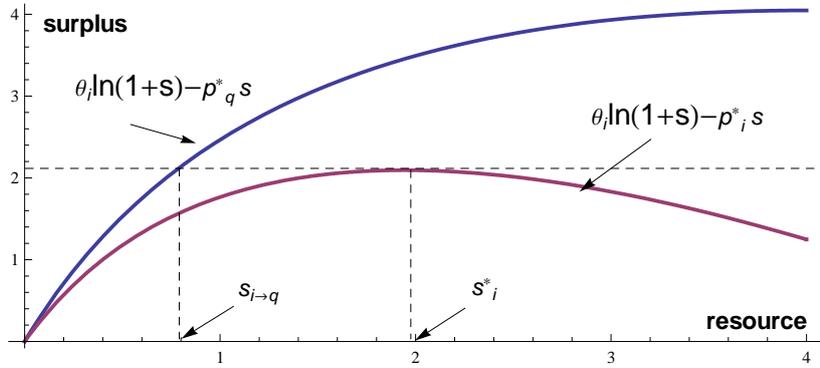


Figure 2.9: When the threshold $s_{th}^{q-1} < s_{i \rightarrow q}$, the group i user can not obtain $U(s_i^*; p_i^*)$ if it chooses the lower price p_q at a quantity less than s_{th}^{q-1} . Therefore it will automatically choose the high price p_i^* to maximize its surplus.

To maintain the group i users' incentive to choose the higher price p_i^* instead of p_q^* , we must have $s_{th}^{q-1} \leq s_{i \rightarrow q}$, which means a group i user can not obtain

$U_i(s_i^*, p_i^*)$ if it chooses a quantity less than s_{th}^{q-1} . In other words, it will automatically choose the higher (and the desirable) price p_i^* to maximize its surplus. On the other hand, we must have $s_{th}^{q-1} \geq s_q^*$ in order to maintain the optimal resource allocation and allow a group q user to choose the right quantity-price combination (illustrated in Fig. 2.8).

Therefore, it is clear that the *necessary and sufficient* condition that the *ICCP* scheme under incomplete information achieves the same maximum revenue of the *CP* scheme under complete information is

$$s_q^* \leq s_{i \rightarrow q}, \quad \forall i < q, \forall q \in \{2, \dots, K\}. \quad (2.32)$$

By solving these inequalities, we can obtain the following theorem (detailed proof in Appendix 2.10.5).

Theorem 2.6. *There exist unique thresholds $\{t_1, \dots, t_{K-1}\}$, such that the *ICCP* scheme achieves the same maximum revenue as in the complete information case if*

$$\sqrt{\frac{\theta_q}{\theta_{q+1}}} \geq t_q \quad \text{for } q=1, \dots, K-1.$$

Moreover, t_q is the unique solution of the equation

$$t^2 \ln t - (t^2 - 1) + \frac{t \sum_{k=1}^q N_k + N_{q+1}}{S + \sum_{k=1}^{K^{cp}} N_k} (t - 1) = 0$$

over the domain $t > 1$.

We want to mention that the condition in Theorem 2.6 is necessary and sufficient for the case of $K = 2$ effective groups³. For $K > 2$, Theorem 2.6 is sufficient but not necessary. The intuition of Theorem 2.6 is that users need to be sufficiently different to achieve the maximum revenue.

The following result immediately follows Theorem 2.6.

³There might be other groups who are not allocated positive resource under the optimal pricing.

Corollary 2.1. *The t_q s in Theorem 2.6 satisfy $t_q < t_{root}$ for $q = 1, \dots, K - 1$, where $t_{root} \approx 2.21846$ is the larger root of equation $t^2 \ln t - (t^2 - 1) = 0$.*

The Corollary 2.1 means that the users do not need to be extremely different to achieve the maximum revenue.

When the conditions in Theorem 2.6 are not satisfied, there may be revenue loss by using the pricing menu in (2.30). Since it is difficult to explicitly solve the parameterized transcend equation (2.31), we are not able to characterize the loss in a closed form yet.

2.6.1 Extensions to Partial Price Differentiation under Incomplete Information

For any given system parameters, we can numerically check whether a partial price differentiation scheme can achieve the same maximum revenue under both the complete and incomplete information scenarios. The idea is similar as we described in this section. Since the *PP* problem can be viewed as the *CP* problem for all effective super-groups, then we can check the *ICCP* bound in Theorem 2.6 for super-groups (once the super-group partition is determined by the searching using Algorithm 4). Deriving an analytical sufficient condition (as in Theorem 2.6) for an incentive compatible partial price differentiation scheme, however, is highly non-trivial and is part of our future study.

2.7 Connections with the Classical Price Differentiation Taxonomy

In economics, price differentiation is often categorized by the first/second/third degree price differentiation taxonomy [40]. This taxonomy is often used in the context of unlimited resources and general pricing functions. The proposed schemes in this chapter have several key differences from these standard concepts, mainly due to the assumption of limited total resources and the choice of linear usage-based pricing.

In the first-degree price differentiation, each user is charged a price based on its willingness to pay. Such a scheme is also called the perfect price differentiation, as it captures users' entire surpluses (*i.e.*, leaving users with zero payoffs). For the complete price differentiation scheme under complete information in Section 2.3, the service provider does not extract all surpluses from users, mainly due to the choice of linear price functions. All effective users obtain positive payoffs.

In the second-degree price differentiation, prices are set according to quantities sold (*e.g.*, the volume discount). The pricing scheme under incomplete information in Section 2.6 has a similar flavor of quantity-based charging. However, our proposed pricing scheme charges a *higher* unit price for a *larger* quantity purchase, which is opposite to the usual practice of volume discount. This is due to our motivation of mimicking the optimal pricing differentiation scheme under the complete information. Our focus is to characterize the sufficient conditions, under which the revenue loss due to incomplete information (also called "information rent" [22–24, 42]) is zero.

In the third-degree price differentiation, prices are set according to some cus-

customer segmentation. The segmentation is usually made based on users' certain attributes such as ages, occupations, and genders. The partial price differentiation scheme in Section 2.5 is analogous to the third-degree price differentiation, but here the user segmentation is still based on users' willingness to pay. The motivation of our scheme is to reduce the implementational complexity.

2.8 Numerical Results

We provide numerical examples to quantitatively study several key properties of price differentiation strategies in this section.

2.8.1 When is price differentiation most beneficial?

Definition 2.1. (*Revenue gain*) We define the revenue gain G of one pricing scheme as the ratio of the revenue difference (between this pricing scheme and the single pricing scheme) normalized by the revenue of single pricing scheme.

In this subsection, we will study the revenue gain of the CP scheme, i.e., $G(\mathbf{N}, \boldsymbol{\theta}, S) \triangleq \frac{R_{cp} - R_{sp}}{R_{sp}}$, where $\mathbf{N} \triangleq \{N_i, \forall i \in \mathcal{I}\}$ denotes the number of users in each groups, $\boldsymbol{\theta} \triangleq \{\theta_i, \forall i \in \mathcal{I}\}$ denotes their willingness to pay, and S is the total resource. Notice that this gain is the maximum possible differentiation gain among all PP schemes.

We first study a simple two-group case. According to Theorems 2.1 and 2.2, the revenue under the SP scheme and the CP scheme can be calculated as follows:

$$R^{sp} = \begin{cases} \frac{S(N_1\theta_1 + N_2\theta_2)}{N_1 + N_2 + S} & 1 \leq t < \sqrt{\frac{S + N_1}{N_1}}, \\ \frac{SN_1\theta_1}{N_1 + S} & t \geq \sqrt{\frac{S + N_1}{N_1}}, \end{cases}$$

and

$$R^{cp} = \begin{cases} \frac{S(N_1\theta_1 + N_2\theta_2) + N_1N_2(\sqrt{\theta_1} - \sqrt{\theta_2})^2}{N_1 + N_2 + S} & 1 \leq t < \frac{S + N_1}{N_1}, \\ \frac{SN_1\theta_1}{N_1 + S} & t \geq \frac{S + N_1}{N_1}. \end{cases}$$

where $t = \sqrt{\frac{\theta_1}{\theta_2}} > 1$.

The revenue gain will depend on five parameters, S , N_1 , θ_1 , N_2 and θ_2 . To simplify notations, let $N = N_1 + N_2$ be the total number of the users, $\alpha = \frac{N_1}{N}$ the percentage of group 1 users, and $\bar{s} = \frac{S}{N}$ the level of normalized available resource.

Thus the revenue gain can be expressed as

$$G(t, \alpha, \bar{s}) = \begin{cases} \frac{\alpha(1-\alpha)(t-1)^2}{\bar{s}(1+\alpha(t^2-1))} & 1 < t < \sqrt{\frac{\bar{s}+\alpha}{\alpha}}, \\ \frac{(1-\alpha)(\bar{s}+\alpha-t\alpha)^2}{\alpha\bar{s}(1+\bar{s})t^2} & \sqrt{\frac{\bar{s}+\alpha}{\alpha}} \leq t \leq \frac{\bar{s}+\alpha}{\alpha}. \end{cases} \quad (2.33)$$

Next we discuss the impact of each parameter.

Observation 2.1. *In terms of the parameter t , G monotonically increases in $(1, \sqrt{\frac{\bar{s}+\alpha}{\alpha}})$ and decrease in $[\sqrt{\frac{k+\alpha}{\alpha}}, \frac{k+\alpha}{\alpha})$. The maximum is obtained at $t_{G-max} = \sqrt{\frac{k+\alpha}{\alpha}}$, when the resource allocated to the group 2 user just becomes zero in the SP scheme.*

One example is showed in Fig.2.10.

It is clear that the revenue gain is not monotonic in the willingness to pay ratio. Its behavior can be divided into three regions: the increasing Region (1) with $t \in (1, \sqrt{\frac{\bar{s}+\alpha}{\alpha}})$, the decreasing Region (2) with $t \in [\sqrt{\frac{k+\alpha}{\alpha}}, \frac{k+\alpha}{\alpha})$, and the zero Region (3) with $t \geq \frac{k+\alpha}{\alpha}$.

It is also interesting to note that three regions are closed related to the effective market sizes: $K^{sp} = K^{cp} = 2$ in Region (1); $K^{sp} = 1$ and $K^{cp} = 2$ in Region (2); and $K^{cp} = K^{sp} = 1$ in Region (3) where the CP scheme degenerates to the SP scheme. The peak point of the revenue gain correspond to the place where the effective market of the SP Scheme changes.

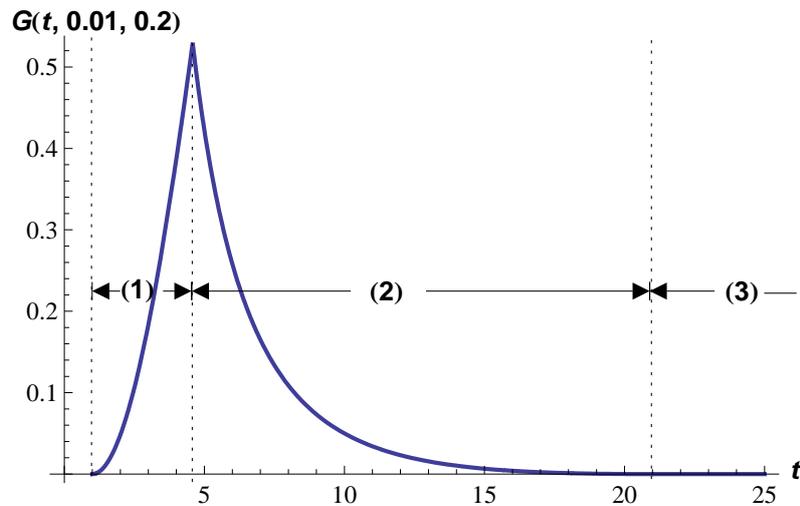


Figure 2.10: One example of the revenue gain $G(t, 0.01, 0.2)$ for the CP scheme. It is clear that the revenue gain can be divided into three regions. Region(1), increasing region, where $K^{cp} = K^{sp} = 2$, and the revenue gain comes from the differentiation gain. Region(2), decreasing region, where $K^{cp} = 2$, $K^{sp} = 1$, and the revenue gain comes from larger effective market and differentiation gain. Region(3), zero region, where $K^{cp} = K^{sp} = 1$, and is a degenerating case where two pricing scheme coincide.

Intuitively, the CP scheme increases the revenue by charging the high willingness to pay groups with high prices, thus the revenue gain increases first when the difference of willingness to pay increases. However, when the difference of willingness to pay is very large, the CP scheme obtain most revenue from the users with high willingness to pay, while the SP scheme declines the users with low willingness to pay but serves the users with high willingness to pay only. Both schemes lead to similar resource allocation in this region, and thus the revenue gain decreases as the difference of willingness to pay increases.

Figure 2.10 shows the revenue gain under usage-based pricing can be very

high in some scenario, e.g., over 50% in this example. We can define this peak revenue gain as

$$G_{max}(\alpha, \bar{s}) = \max_{t \geq 1} G(t, \alpha, \bar{s}) = \frac{(\alpha - 1)(\sqrt{\bar{s} + \alpha} - \sqrt{\alpha})^2}{\bar{s}(1 + \bar{s})}.$$

Figure 2.11 is shown how G_{max} changes in \bar{s} with different parameters α .

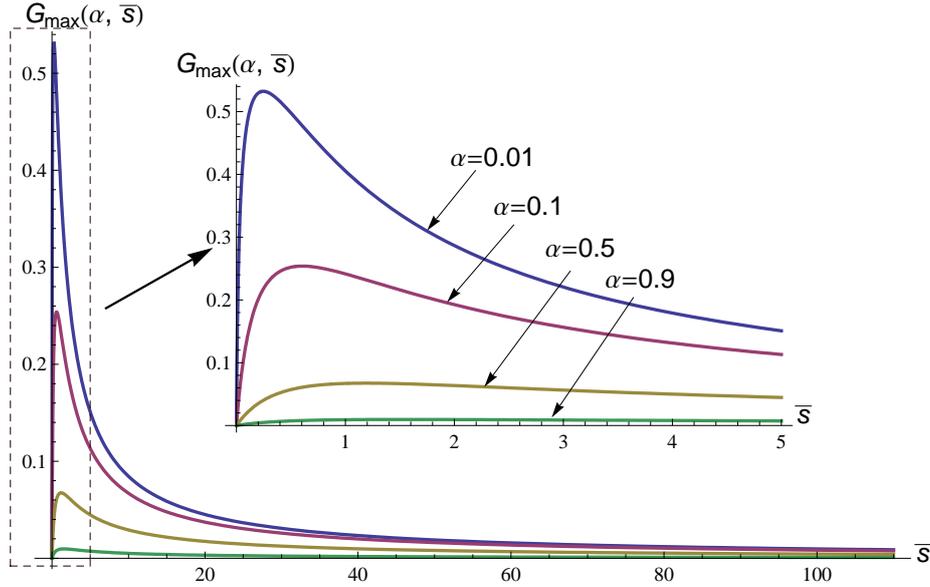


Figure 2.11: For a fixed \bar{s} , $G_{max}(\alpha, \bar{s})$ monotonically increases in α . For a fixed α , $G_{max}(\alpha, \bar{s})$ first increases in \bar{s} , and then decreases in \bar{s}

Observation 2.2. For a fixed \bar{s} , $G_{max}(\alpha, \bar{s})$ monotonically decreases in α .

When α is small, which means users with high willingness to pay are minorities in the effective market, the advantage of price differentiation is very evident. As shown in Fig. 2.11, when $\alpha = 0.1$, the maximum possible revenue gain can be over than 20%; and when $\alpha = 0.01$, this gain can be even higher than 50%. However, when users with high willingness to pay are majority, the price differentiation gain is very limited, for example, the gain is no larger than 8% and 2% for $\alpha = 0.5$ and 0.9, respectively.

Intuitively, users with high willingness to pay are the most profitable users in the market. Ignoring them is detrimental in terms of revenue even if they only occupy a small fraction of the population. Since the SP scheme is set based on the average willingness to pay of the effective market, the users with high willingness to pay will be ignored (in the sense of not charging the desirable high price) when α is small. In contrast, ignoring the users with low willingness to pay when α is large is not a big issue.

Observation 2.3. *For parameter k , $G_{max}(\alpha, \bar{s})$ is not a monotonic function in \bar{s} . Its shape looks like a skewed bell. The gain is either small when \bar{s} is very small or very large.*

Small \bar{s} means that resource is very limited, and both schemes allocates the resource to users with high willingness to pay (see the discussion of the threshold structure in Sections 2.3 and 2.4), and thus there is not much difference between two pricing schemes. While \bar{s} is very large, i.e., the resource is abundant, the prices and the resource allocation with or without differentiation become similar (which can be easily checked from formulations in Theorems 2.1 and 2.2). In these two scenarios, similar resource allocations lead to similar revenues. These explains the bell shape for parameter \bar{s} .

Based on the above observations, we find that the revenue gain can be very high under two conditions. First, the users with high willingness to pay are minorities in the effective market. Second, the total resource is comparatively limited.

For cases with three or more groups, the analytical study becomes much more challenging due to many more parameters. Moreover, the complex threshold structure of the effective market makes the problem even complicated. We will present some numerical studies to illustrate some interesting insights.

Table 2.2: Parameter settings of a three-group example

	θ_1	N_1	θ_2	N_2	θ_3	N_3	$\bar{\theta}$
Case 1	9	10	3	10	1	80	2
Case 2	3	33	2	33	1	34	2
Case 3	2.2	80	1.5	10	1	10	2

For illustration convenience, we choose a three-group example and three different sets of parameters as shown in Table 2.2. To limit the dimension of the problem, we set the parameters such that the total number of users and the average willingness to pay (i.e., $\bar{\theta} = \sum_{i=1}^3 N_i \theta_i / (\sum_{i=1}^3 N_i)$) of all users are the same across three different parameter settings. This ensures that the *SP* scheme achieves the same revenue in three different cases when resource is abundant. Figure 2.12 illustrates how the differentiation gain changing changes in resource S .

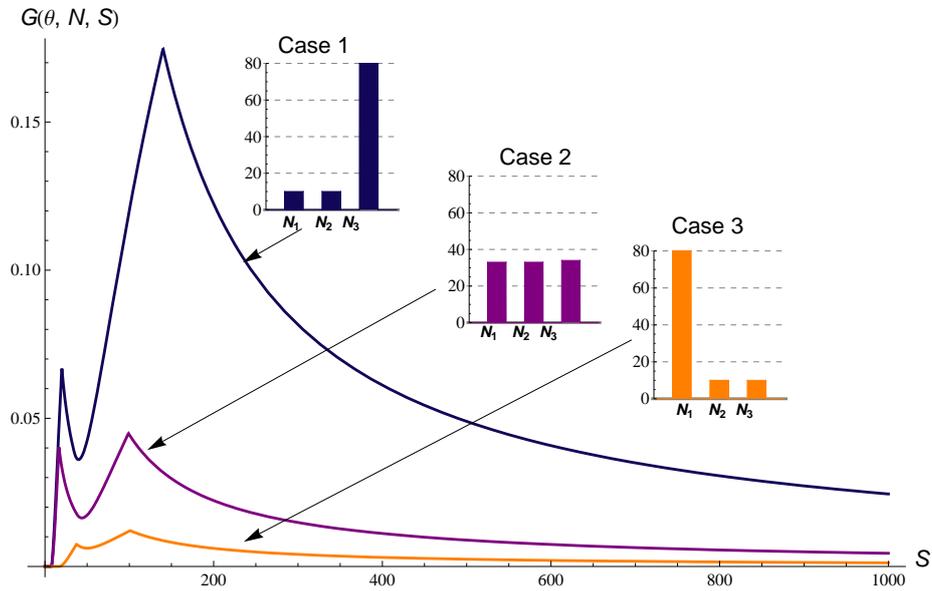


Figure 2.12: An example of the revenue gain of the three-group market with the same average willingness to pay

Similar as the analytical study of the two-group case, Fig. 2.12 shows that the revenue gain is large only when the users with high willingness to pay are minorities (e.g. case 1) in the effective market and the resource is limited but not too small ($100 \leq S \leq 150$ in all three cases). When resource S is large enough (e.g., ≥ 150), the gain will gradually diminish to zero as the resource increases. For each curve in Fig. 2.12, there are two peak points. Each peak point represents a change of the effective market threshold in the SP scheme, i.e., when the resource allocation to a group becomes zero. In numerical studies of networks with $I > 3$ groups (not shown in this chapter), we have observed the similar conditions for achieving a large differentiation gain and the phenomenon of $I - 1$ peak points.

2.8.2 What is the best tradeoff of Partial Price Differentiation?

In Section 2.5, we design Algorithm 4 that optimally solves the PP problem with a polynomial complexity. Here we study the tradeoff between total revenue and implementational complexity.

To illustrate the tradeoff, we consider a five-group example with parameters shown in Table 2.3. Note that users with high willingness to pay are minorities here. Figure 2.13 shows the revenue gain G as a function of total resource S under different PP schemes (including CP scheme as a special case), and Fig. 2.14 shows how the effective market thresholds change with the total resource.

Table 2.3: Parameter setting of a five-group example

group index i	1	2	3	4	5
θ_i	16	8	4	2	1
N_i	2	3	5	10	80

We enlarge Fig. 2.13 and Fig. 2.14 within the range of $S \in [0, 50]$, which is the most complex and interesting part due to several peak points. Similar as Fig. 2.12, we observe $I - 1 = 4$ peak points for each curve in Fig. 2.13. Each peak point again represents a change of effective market threshold of the single pricing scheme, as we can easily verify by comparing Fig. 2.14 with Fig. 2.13.

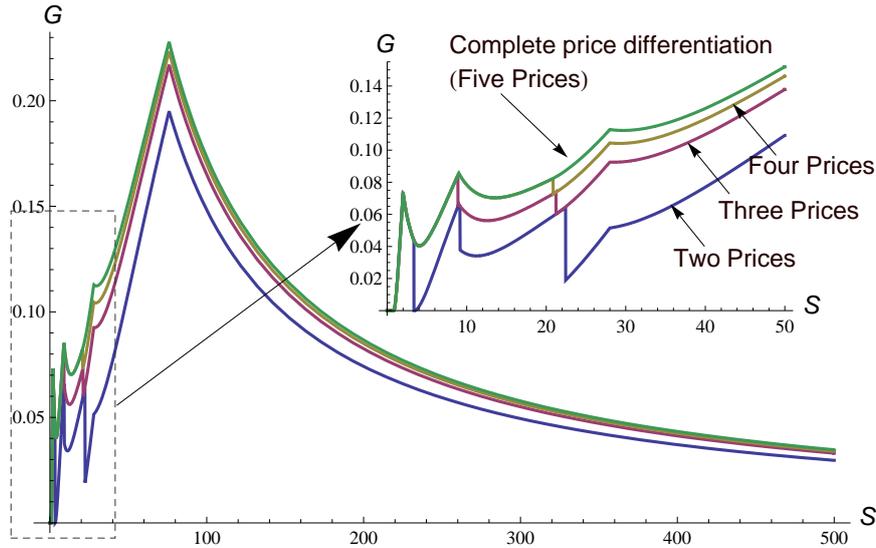


Figure 2.13: Revenue gain of a five-group example under different price differentiation schemes

As the resource S increases from 0, all gains in Fig. 2.13 first overlap with each other, then the two-price scheme (blue curve) separates from the others at $S = 3.41$, after that the three-price scheme (purple curve) separates at $S = 8.89$, and finally the four-price scheme (dark yellow curve) separates at near $S = 20.84$. These phenomena are due to the threshold structure of the PP scheme. When the resource is very limited, the effective markets under all pricing scheme include only one group with the highest willingness to pay, and all pricing schemes coincide with the SP scheme. As the resource increases, the effective market enlarges from two groups to finally five groups. The change of the effective market thresh-

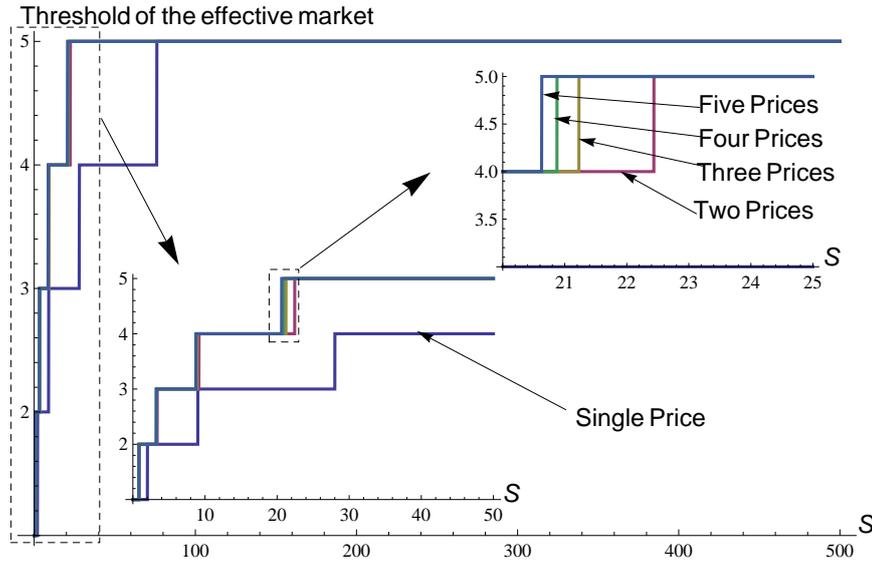


Figure 2.14: Corresponding thresholds of effective markets of Fig. 2.13's example

old can be directly observed in Fig. 2.14. Comparing across different curves in Fig. 2.14, we find that the effective market size is non-decreasing with the number of prices for the same resource S . This agrees with our intuition in Section 2.4.2, which states that the size of effective market indicates the degree of differentiation.

Figure 2.13 provides the service provider a global picture of choosing the most proper pricing scheme according to achieve the desirable financial target under a certain parameter setting. For example, if the total resource $S = 100$, the two-price scheme seems to be a sweet spot, as it achieves a differential gain of 14.8% comparing to the SP scheme and is only 2.4% worse than the CP scheme with five prices.

2.9 Summary

In this chapter, we study the revenue-maximizing problem for a monopoly service provider under both complete and incomplete network information. Under

complete information, our focus is to investigate the tradeoff between the total revenue and the implementational complexity (measured in the number of pricing choices available for users). Among the three pricing differentiation schemes we proposed (*i.e.*, complete, single, and partial), the partial price differentiation is the most general one and includes the other two as special cases. By exploiting the unique problem structure, we designed an algorithm that computes the optimal partial pricing scheme in polynomial time, and numerically quantize the tradeoff between implementational complexity and total revenue. Under incomplete information, designing an incentive-compatible differentiation pricing scheme is difficult in general. We show that when the users are significantly different, it is possible to design a quantity-based pricing scheme that achieves the same maximum revenue as under complete information.

2.10 Appendix of Chapter 2

2.10.1 Complete Price Differentiation under complete information with General Utility Functions

In this section, we extend the solution of the complete price differentiation problem to general form of increasing and concave utility functions $u_i(s_i)$. We denote $R_i(s_i)$ as the revenue collected from one user in group i . Based on the stackelberg model, the prices satisfy $p_i = u'_i(s_i)$, $s_i \geq 0$ $i \in \mathcal{I}$, thus

$$R_i(s_i) = u'_i(s_i)s_i, s_i \geq 0. \quad (2.34)$$

Therefore, we can rewrite the Complete Price differentiation problem with

General utility function (*CPG*) as follows.

$$\begin{aligned} CPG : \quad & \underset{s \geq 0, \mathbf{n}}{\text{maximize}} && \sum_{i \in \mathcal{I}} n_i R_i(s_i) \\ & \text{subject to} && n_i \in \{0, \dots, N_i\} \quad , \quad i \in \mathcal{I} \end{aligned} \quad (2.35)$$

$$\sum_{i \in \mathcal{I}} n_i s_i \leq S \quad (2.36)$$

By similar solving technique in Sec. 2.3, we can solve the *CPG* problem by decomposing it into two subproblems: the resource allocation subproblem CPG_1 , and the admission control subproblem CPG_2 . In the subproblem CPG_1 , for given \mathbf{n} , we solve

$$\begin{aligned} CPG_1 : \quad & \underset{s \geq 0}{\text{maximize}} && \sum_{i \in \mathcal{I}} n_i R_i(s_i) \\ & \text{subject to} && \sum_{i \in \mathcal{I}} n_i s_i \leq S \end{aligned}$$

After solving the optimal resource allocation $s_i^*(\mathbf{n})$, $i \in \mathcal{I}$, we further solve the admission control subproblem:

$$\begin{aligned} CPG_2 : \quad & \underset{\mathbf{n}}{\text{maximize}} && \sum_{i \in \mathcal{I}} n_i R_i(s_i^*(\mathbf{n})) \\ & \text{subject to} : && n_i \in \{0, \dots, N_i\}. \end{aligned}$$

We are especially interested in the case that constraint (2.36) is active in the *CPG* problem, which means the resource bound is tight in the considered problem; otherwise, the *CPG* problem degenerates to a revenue maximization without any bounded resource constraint. We can prove the following results.

Proposition 2.2. *If the resource constraint (2.36) is active in the optimal solution of the *CPG* problem (or the CPG_1 subproblem), then one of optimal solutions of the CPG_2 subproblem is*

$$n_i^* = N_i, \quad i \in \mathcal{I}. \quad (2.37)$$

Proof. We first release the variable n_i to real number, and calculate the first derivative as follows:

$$\frac{\partial R_i}{\partial n_i} = R_i(s_i^*) + n_i \frac{\partial R_i(s_i^*)}{\partial s_i} \frac{\partial s_i^*}{\partial n_i}, \quad i \in \mathcal{I}. \quad (2.38)$$

Plugging (2.34), $R'_i(s_i) = u''_i(s_i) s_i + u'_i(s_i)$, and we have

$$\frac{\partial R_i}{\partial n_i} = u'_i(s_i^*) \left(s_i^* + n_i \frac{\partial s_i^*}{\partial n_i} \right) + n_i u''_i(s_i^*) s_i^* \frac{\partial s_i^*}{\partial n_i}, \quad i \in \mathcal{I}. \quad (2.39)$$

Since the resource constraint (2.36) is active in the optimal solution of the CPG_1 subproblem, that is, $\sum_{i \in \mathcal{I}} n_i s_i = S$, by taking derivative of n_i in both sides of it, we have:

$$s_i^* + n_i \frac{\partial s_i^*}{\partial n_i} = 0. \quad (2.40)$$

Substituting (2.40) into (2.39), since we assume the utility function $u_i(s_i)$ is increasing and concave function, then we have

$$\frac{\partial R_i}{\partial n_i} = -u''_i(s_i^*) s_i^{*2} \geq 0, \quad i \in \mathcal{K}. \quad (2.41)$$

Thus we can conclude that one of optimal solutions for the CPG_2 subproblem is $n_i^* = N_i, i \in \mathcal{I}$. \square

Proposition 2.2 points out that when the resource constraint (2.36) is active, the CPG problem can be greatly simplified: its solution can be obtained by solving the CPG subproblem with parameters $n_i = N_i, i = 1, \dots, I$. The following proposition provides a sufficient condition that the resource constraint (2.36) is active.

Proposition 2.3. *If $u''_i(s_i) s_i + u_i(s_i) > 0, s_i \geq 0, i \in \mathcal{I}$, then the resource constraint is active at the optimal solution.*

Proof. Let λ and $\mu_i, i \in \mathcal{I}$, be the Lagrange multiplier of constraint (2.36) and $s_i \geq 0, i \in \mathcal{I}$ respectively, thus the KKT conditions of the CGP_1 subproblem is

given as follows:

$$\begin{aligned} n_i \frac{\partial R_i(s_i^*)}{\partial s_i} - n_i \lambda^* + \mu_i^* &= 0, \quad i \in \mathcal{I}; \\ \lambda^* \left(\sum_{i \in \mathcal{I}} n_i s_i^* - S \right) &= 0; \\ \mu_i^* s_i^* &= 0; \\ \lambda^* &\geq 0; \\ \mu_i^* &\geq 0, \quad i \in \mathcal{I}; \\ s_i^* &\geq 0, \quad i \in \mathcal{I}. \end{aligned}$$

We denote $\mathcal{K} := \{i \mid s_i^* > 0\}$, and $\bar{\mathcal{K}} := \{i \mid s_i^* = 0\}$.

For $i \in \mathcal{K}$:

$$\frac{\partial R_i(s_i^*)}{\partial s_i} = \lambda^*, \quad i \in \mathcal{I}; \quad (2.42)$$

$$\lambda^* \left(\sum_{i \in \mathcal{K}} n_i s_i^* - S \right) = 0. \quad (2.43)$$

For $i \in \bar{\mathcal{K}}$:

$$\frac{\partial R_i(0)}{\partial s_i} \leq \lambda^*, \quad i \in \mathcal{I}; \quad (2.44)$$

Since $u_i''(s_i)s_i + u_i(s_i) > 0$, $s_i \geq 0$, $i \in \mathcal{I}$ and (2.42), we have

$$\lambda^* = \frac{\partial R_i(s_i^*)}{\partial s_i} = u_i''(s_i^*)s_i^* + u_i(s_i^*) > 0.$$

By (2.43), we must have $\sum_{i \in \mathcal{I}} n_i s_i^* - S = 0$, that the resource constraint is active at the optimal solution. \square

Next, let us discuss how to calculate the optimal solution. To guarantee uniqueness resource allocation solution, we assume that revenue in is a strictly concave function of the demand⁴, *i.e.*, $\frac{\partial^2 R_i(s_i)}{\partial s_i^2} < 0$, $i \in \mathcal{I}$. Thus we have the following theorem.

⁴This assumption has been frequently used in the revenue management literature [43].

Theorem 2.7. *If $\frac{\partial^2 R_i(s_i)}{\partial s_i^2} < 0$, $i \in \mathcal{I}$, then there exists an optimal solution of the CGP problem as follows:*

- *All users are admitted: $n_i^* = N_i$ for all $i \in \mathcal{I}$.*
- *There exist a value λ^* and a group index threshold $K^{cp} \leq I$, such that only the top K^{cp} groups of users receive positive resource allocations,*

$$s_i^* = \begin{cases} \frac{\partial R_i}{\partial s_i}^{-1}(\lambda^*), & i \in \mathcal{K}; \\ 0, & \text{otherwise.} \end{cases} \quad (2.45)$$

where values of λ^* and effective market \mathcal{K} can be computed as in Algorithm 5.

In Algorithm 5, we use notation f^{-1} denotes its inverse function, and rearrange the group index satisfying $\frac{\partial R_{(1)}}{\partial s_{(1)}}^{-1}(0) \geq \frac{\partial R_{(2)}}{\partial s_{(2)}}^{-1}(0) \geq \dots \geq \frac{\partial R_{(I)}}{\partial s_{(I)}}^{-1}(0)$.

Algorithm 5 Search the threshold for general utility function

- 1: $k \leftarrow I, \lambda \leftarrow \frac{\partial R_{(k)}}{\partial s_{(k)}}^{-1}(0)$
 - 2: **while** $\sum_{i=1}^k n_{(i)} \left(\frac{\partial R_{(i)}}{\partial s_{(i)}}^{-1}(\lambda) \right)^+ \geq S$, **do**
 - 3: $k \leftarrow k - 1$
 - 4: $\lambda \leftarrow \frac{\partial R_{(k)}}{\partial s_{(k)}}^{-1}(0)$
 - 5: **end while**
 - 6: **Return** $\mathcal{K} = \{(1), (2), \dots, (k)\}$
-

Remark 2.2. *The complexity of Algorithm 5 is also $\mathcal{O}(I)$, i.e., linear in the number of user groups (not the number of users).*

Remark 2.3. *There are several functions satisfying the technical conditions in Theorem 2.7, e.g., the standard α -fairness functions*

$$u_i(s_i) = \begin{cases} (1 - \alpha)^{-1} s_i^{1-\alpha}, & 0 \leq \alpha < 1; \\ \log s_i, & \alpha = 1. \end{cases}$$

2.10.2 Proof of Proposition 2.1

Proof. We first focus on the key water-filling problems that we solve for the two pricing schemes (the CP scheme on the LHS and the SP scheme on the RHS):

$$\sum_{i \in \mathcal{I}} N_i \left(\sqrt{\frac{\theta_i}{\lambda^*}} - 1 \right)^+ = S = \sum_{i \in \mathcal{I}} N_i \left(\frac{\theta_i}{p^*} - 1 \right)^+ \quad (2.46)$$

Let $\theta = \frac{p^{*2}}{\lambda^*}$ be the solution of the equation of $\sqrt{\frac{\theta}{\lambda^*}} = \frac{\theta}{p^*}$. By comparing it with $\theta_i, i \in \mathcal{I}$, there are three cases:

- Case 1:

$$\theta > \theta_1 \Rightarrow \sqrt{\frac{\theta_i}{\lambda^*}} = \frac{\sqrt{\theta_i} \sqrt{\theta}}{p^*} > \frac{\theta_i}{p^*}, \forall i \in \mathcal{I}.$$

This case can not be possible. Since if every term in the left summation is strictly larger than its counterpart in the right summation, then (2.46) can not hold.

- Case 2: $\theta_I \geq \theta \Rightarrow \sqrt{\frac{\theta_i}{\lambda^*}} = \frac{\sqrt{\theta_i} \sqrt{\theta}}{p^*} \leq \frac{\theta_i}{p^*}, \forall i \in \mathcal{I}$. Similarly as Case 1, it can not hold, either.

- Case 3: $\exists k, \text{ s.t. } 1 \leq k < I$ and $\theta_k \geq \theta \geq \theta_{k+1}$

$$\Rightarrow \left\{ \begin{array}{l} \sqrt{\frac{\theta_i}{\lambda^*}} = \frac{\sqrt{\theta_i} \sqrt{\theta}}{p^*} \leq \frac{\theta_i}{p^*}, i = 1, 2, \dots, k; \\ \text{The equality holds only when } \theta = \theta_k \text{ and } i = k. \\ \\ \sqrt{\frac{\theta_i}{\lambda^*}} = \frac{\sqrt{\theta_i} \sqrt{\theta}}{p^*} \geq \frac{\theta_i}{p^*}, i = k + 1, \dots, I. \\ \text{The equality holds only when } \theta = \theta_{k+1} \text{ and } i = k + 1. \end{array} \right.$$

Similar argument as the above two case, we have $K^{cp} \geq k$ and $K^{sp} \geq k$, otherwise (2.46) can not hold. Further, $K^{cp} \geq K^{sp}$, since if $\frac{\theta_{K^{sp}}}{p^*} - 1 > 0$, then $\sqrt{\frac{\theta_{K^{cp}}}{\lambda^*}} - 1 > 0$.

By Theorems 2.1 and 2.2, we prove the proposition. \square

2.10.3 Proof of Lemma 2.2

We can first prove the following lemma.

Lemma 2.3. *Suppose an effective market of the single pricing scheme is denoted as $\mathcal{K} = \{1, 2, \dots, K\}$. If we add a new group v of N_v users with $\theta_v > \theta_K$, then the revenue strictly increases.*

Proof. We denote the single price before joining group v is p , the price after joining group v is p' , the effective market become \mathcal{K}' . By Theorem 2.2, we have

$$p = \frac{\sum_{i=1}^K N_i \theta_i}{S + \sum_{i=1}^K N_i} \text{ with } \theta_K > p \text{ and } \theta_{K+1} \leq p.$$

Since the optimal revenue is obtained by selling out the total resource S , thus to prove the total revenue strictly increases if and only if we can prove $p' > p$. We consider the following two cases.

- If after group v joining in, the new effective market satisfies $\mathcal{K}' = \mathcal{K} \cup \{v\}$, then we have

$$p' = \frac{\sum_{i=1}^K N_i \theta_i + N_v \theta_v}{S + \sum_{i=1}^K N_i + N_v}.$$

Since $\theta_v > \theta_K > p$, we have $p' > p$, due to the following simple fact.

Fact 2.1. *For any $a_1, b_1, a_2, b_2 > 0$, the following two inequality are equivalent:*

$$\frac{a_1}{b_1} \geq \frac{a_2}{b_2} \Leftrightarrow \frac{a_1}{b_1} \geq \frac{a_1 + a_2}{b_1 + b_2} \geq \frac{a_2}{b_2}. \quad (2.47)$$

- If after group v joining in, the new effective market shrinks, namely, $\mathcal{K}' \subset \mathcal{K} \cup \{v\}$, $\mathcal{K}' \neq \mathcal{K} \cup \{v\}$, then we have $p' > \theta_K > p$.

□

By the above Lemma 2.3, we further prove Lemma 2.2.

Proof. We prove Lemma 2.2 by contradiction. Suppose that the group indices of the effective market under the optimal partition α is not consecutive. Suppose that group i is not an effective group, and there exists some group $j, j > i$, which is an effective group. We consider a new partition α' by putting group i into the cluster to which group j belongs, and keeping other groups unchanged. According to Lemma 2.3, the revenue under partition α' is greater than that under partition α , thus partition α is not optimal. This contradicts to our assumption and thus completes the proof. \square

2.10.4 Proof of Theorem 2.4

For convenience, we use the notation $(\dots \cup \dots | \dots \cup \dots | \dots)$ to denote a partition with the groups between bars connected with “ \cup ” representing a cluster, e.g., three partitions for $J = 2, K^{pp} = 3$ are $(1|2 \cup 3), (1 \cup 2 | 3)$ and $(1 \cup 3 | 2)$. In addition, we introduce the *compound group* to simplify the notation of complex clusters with multiple groups. A cluster containing group i can be simply represented as $Pre(i) \cup i \cup Post(i)$, where $Pre(i)$ (or $Post(i)$) refers as a compound group composing of all the groups with willingness to pay larger (or smaller) than that of group i in the cluster. Note that the compound groups can be empty in certain cases.

Before we prove the general case in Theorem 2.4, we first prove the results is true for the following two special cases in Lemma 2.4 and Lemma 2.5.

Lemma 2.4. *For a three-group effective market with two prices, i.e., $K^{pp} = 3, J = 2$, an optimal partition involves consecutive group indices within clusters.*

Proof. There are three partitions for $K^{pp} = 3, J = 2$, and only $(1 \cup 3 | 2)$ is with discontinuous group index within clusters. To show our result, we only need to prove one of partitions with group consecutive is better than $(1 \cup 3 | 2)$. We have

two main steps in this proof, first we prove this result is true for the PP problem without consider Constraint (2.29). Further, we show that Constraint (2.29) will not affect the optimality of partitions with consecutive group indices within each cluster.

Step 1: (Without Constraint (2.29))

Without considering Constraint (2.29), we want show that $\mathbf{a}_1 = (1 \cup 2 | 3)$ is always better than $\mathbf{a}_2 = (1 \cup 3 | 2)$. Mathematically, what we try to prove is:

$$v(\mathbf{a}_2) > v(\mathbf{a}_1). \quad (2.48)$$

where $v(\mathbf{a}_2) = (N_1 + N_3) \sqrt{\frac{N_1\theta_1 + N_3\theta_3}{N_1 + N_3}} + N_2\sqrt{\theta_2}$, and $v(\mathbf{a}_1) = (N_1 + N_2) \sqrt{\frac{N_1\theta_1 + N_2\theta_2}{N_1 + N_2}} + N_3\sqrt{\theta_3}$. With the new notation

$$\Delta V(i, j) := (N_i + N_j) \sqrt{\frac{N_i\theta_i + N_j\theta_j}{N_i + N_j}} - N_i\sqrt{\theta_i} - N_j\sqrt{\theta_j},$$

it is easy to see that (2.48) is equivalent to the following inequality:

$$\Delta V(1, 3) > \Delta V(1, 2). \quad (2.49)$$

We prove the inequality (2.49) by considering the following two cases.

a) If $N_1 \leq N_2$, we define a function of x as follows,

$$g(j; x) := (N_j + N_1) \sqrt{\frac{N_j\theta_j + N_1(\theta_j + x)}{N_j + N_1}} - N_1\sqrt{\theta_j + x} - N_j\sqrt{\theta_j}.$$

It is easy to check that

$$g(j; x)|_{x=\theta_1-\theta_j} = \Delta V(1, j), \text{ and } g(j; x)|_{x=0} = 0;$$

and if $x > 0$, then

$$g'(j; x) = \frac{\partial g(j; x)}{\partial x} = \frac{N_1}{2} \left(\frac{1}{\sqrt{\frac{N_j\theta_j + N_1(\theta_j + x)}{N_j + N_1}}} - \frac{1}{\sqrt{\theta_j + x}} \right) > 0,$$

$$\text{and } \frac{\partial g'(j; x)}{\partial \theta_j} = \frac{N_1}{4} \left(\frac{1}{(\theta_j + x)^{1.5}} - \frac{1}{\left(\frac{N_j \theta_j + N_1(\theta_j + x)}{N_j + N_1} \right)^{1.5}} \right) < 0.$$

Since $\theta_2 > \theta_3$, it immediately follows that

$$g'(3; x) \geq \frac{N_1}{2} \left(\frac{1}{\sqrt{\frac{N_3 \theta_2 + N_1(\theta_2 + x)}{N_1 + N_3}}} - \frac{1}{\sqrt{\theta_2 + x}} \right),$$

Since $N_2 \geq N_1$, then we have

$$g'(3; x) \geq \frac{N_1}{2} \left(\frac{1}{\sqrt{\frac{N_3 \theta_2 + N_1(\theta_2 + x)}{N_1 + N_3}}} - \frac{1}{\sqrt{\theta_2 + x}} \right) \geq g'(2; x),$$

Thus, it follows

$$\Delta V(1, 3) = \int_0^{\theta_1 - \theta_3} g'(3; x) dx > \int_0^{\theta_1 - \theta_2} g'(2; x) dx = \Delta V(1, 2),$$

i.e., (2.49) is obtained.

Let us see a special case of (2.49). When $N_1 = N_2$, then

$$\Delta V(1, 2) = (N_1 + N_1) \sqrt{\frac{N_1 \theta_2 + N_1 \theta_1}{N_1 + N_1}} - N_1 \sqrt{\theta_1} - N_1 \sqrt{\theta_2},$$

then we have

$$\Delta V(1, 3) > (N_1 + N_1) \sqrt{\frac{N_1 \theta_2 + N_1 \theta_1}{N_1 + N_1}} - N_1 \sqrt{\theta_1} - N_1 \sqrt{\theta_2}. \quad (2.50)$$

Notice that although (2.50) is defined with the assumption that $N_1 \leq N_2$, it also holds for the case $N_1 > N_2$ as (2.50) does not contain the parameter N_2 . This result will be used in the proof later.

b) If $N_1 > N_2$, we define a function of m as

$$f(m) := (N_1 + m) \sqrt{\frac{N_1 \theta_1 + m \theta_2}{N_1 + m}} - N_1 \sqrt{\theta_1} - m \sqrt{\theta_2}.$$

It is easy to obtain that

$$\frac{df(m)}{dm} = \frac{\left(\sqrt{\frac{N_1 \theta_1 + m \theta_2}{N_1 + m}} - \sqrt{\theta_2} \right)^2}{2 \sqrt{\frac{N_1 \theta_1 + m \theta_2}{N_1 + m}}} > 0,$$

i.e., the function f is an increasing function of m .

Thus it follows that

$$\Delta V(1, 2) = f(N_2) < f(N_1) \stackrel{(a)}{<} \Delta V(1, 3),$$

where (a) results from (2.50), the right hand side of which is equal to $f(N_1)$.

Step 2: (Checking Constraint (2.29))

We want to prove that \mathbf{a}_1 satisfying Constraint (2.29) is the sufficient condition of \mathbf{a}_2 satisfying (2.29).

Consider if \mathbf{a}_1 does not satisfy (2.29), it means

$$\sqrt{\theta_3} \leq \sqrt{\lambda(\mathbf{a}_1)} = \sqrt{\frac{v(\mathbf{a}_1)}{S + \sum_{i=1}^3 N_i}}.$$

By the result in Step 1, we know that $v(\mathbf{a}_1) < v(\mathbf{a}_2)$, then we have

$$\sqrt{\theta_3} < \sqrt{\lambda(\mathbf{a}_2)} = \sqrt{\frac{v(\mathbf{a}_2)}{S + \sum_{i=1}^3 N_i}},$$

and further

$$\theta_3 < \sqrt{\theta_3 \lambda(\mathbf{a}_2)} < \sqrt{\frac{N_3 \theta_3 + N_1 \theta_1}{N_1 + N_3} \lambda(\mathbf{a}_2)} = \sqrt{\theta^1 \lambda(\mathbf{a}_2)}.$$

It means \mathbf{a}_2 can not satisfy (2.29) either. Thus we see that constraint (2.29) actually does not affect the result in Step 1. In conclusion, we show that in a simple case with $K^{pp} = 3$, $J = 2$, an optimal partition involves consecutive group indices within clusters. \square

Further, based on Lemma 2.4 we prove another simple special case.

Lemma 2.5. *For a four-group effective market with two prices, i.e., $K^{pp} = 4$, $J = 2$, an optimal partition involves consecutive group indices within clusters.*

Proof. For $K^{pp} = 4$ and $J = 2$ case, there are total seven possible partitions. Three among them are with consecutive group index, $(1 | 2 \cup 3 \cup 4)$, $(1 \cup 2 | 3 \cup 4)$

and $(1 \cup 2 \cup 3 | 4)$. We denote a set composed by these three partitions as Σ_c . We need to show the remaining four partitions are no better than some partition in Σ_c . To show this, we only need to transform them to some three-group case and apply the result of Lemma 2.4.

- *Case 1:* $(1 \cup 4, 2 \cup 3)$ is not optimal since we can prove $(1 \cup 2 \cup 3, 4) \in \Sigma_c$ is better. To show it, we take $2 \cup 3$ as a whole, then by Lemma 2.4, it follows that $\Delta V(1, 4) > \Delta V(1, 2 \cup 3)$.
- *Case 2:* $(2, 1 \cup 3 \cup 4)$ is not optimal, since we can prove $(1 \cup 2, 3 \cup 4) \in \Sigma_c$ is better. To show it, we take $3 \cup 4$ as a whole, then by Lemma 2.4, it follows $\Delta V(1, 3 \cup 4) > \Delta V(1, 2)$.
- *Case 3:* $(3, 1 \cup 2 \cup 4)$ is not optimal, since we can prove $(1 \cup 2 \cup 3, 4) \in \Sigma_c$ is better. To show it, we take $1 \cup 2$ as a whole, then by Lemma 2.4, it follows that $\Delta V(1 \cup 2, 4) > \Delta V(1 \cup 2, 3)$.
- *Case 4:* $(1 \cup 3, 2 \cup 4)$ is not optimal, since we can prove $(1 \cup 2 \cup 3, 4) \in \Sigma_c$ is better. To show it, by Lemma 2.4, it follows that $\Delta V(2, 4) > \Delta V(2, 3)$, and that $\Delta V(1, 3) \stackrel{(b)}{>} \Delta V(1, 2 \cup 3)$. Here inequality (b) is also easily obtained, if we notice that $\theta_1 > \theta_{2 \cup 3} > \theta_3$, thus group $2 \cup 3$ can be also treated as the role of group 2 in Lemma 2.4.

□

Now Let us prove Theorem 2.4. For convenience, we introduce the notation *Compound group*, such as $Pre(i)$ or $Post(i)$, which represents some part of a cluster with ordered group indices. For a group i in some cluster, $Pre(i)$ (or $Post(i)$) refers as a compound group composing of all the groups with willingness to pay larger (or smaller) than that of group i . For example, in a cluster $1 \cup 2 \cup 3 \cup 5 \cup 7 \cup 8$, $Pre(3) = 1 \cup 2$, $Post(3) = 5 \cup 7 \cup 8$. Note that compound groups can be empty, denoted as \emptyset . In last example, $Pre(1) = Post(8) = \emptyset$. Since

all the groups within the compound group belong to one cluster, we can apply Lemma 2.2. For example, with the previous cluster setting, $N_{Pre(3)} = N_1 + N_2$, and $\theta_{Pre(3)} = \frac{N_1\theta_1 + N_2\theta_2}{N_1 + N_2}$. By this equivalence rule, a compound group actually has not much difference with one original group. The conclusions of Lemma 2.4 and Lemma 2.5 can be easily extended to compound groups.

Proof. Without loss of generality, suppose that the group indices order within each cluster is increasing.

Now consider one partition with discontinuous group indices within some clusters. We can check the group indices continuity for every single group. For example, a group c belonging to a cluster \mathcal{C} , and its next neighbor in this cluster is group d , if $c - d = 1$, then the group indices until c are consecutive, and if $c - d > 1$, then the group indices are discontinuous, and we find a gap between c and d .

Suppose that checking group indices continuity for each group following the increasing indices order (or equivalently decreasing willingness to pay order) from group 1 to group K^{pp} . We do not find any gap until group u_1 in cluster \mathcal{U} . We denote group u_1 next neighbor in cluster \mathcal{U} is group u_2 . Since there is a gap between u_1 and u_2 , there exists a group v in another cluster \mathcal{V} and satisfying $v = u_1 + 1 < u_2$. Now we can construct a better partition by rearranging the two clusters \mathcal{U} and \mathcal{V} , while keeping other clusters unchanged. We can view \mathcal{U} as $(Pre(u_2) \cup Post(u_1))$, and \mathcal{V} as $(v \cup Post(v))$, since there is no group before v in super-group \mathcal{V} , otherwise it contradicts with the fact that we do not find any gap until group u_1 . It is easy to show that there is some new partition better than the original one by Lemma 2.4 and Lemma 2.5. There are two cases depending on whether $Post(v)$ is empty or not. If $Post(v) = \emptyset$, according to Lemma 2.4, we find another partition with $\mathcal{U}' = Pre(u_2) \cup v$, $\mathcal{V}' = Post(u_1)$ better than the original \mathcal{U} and \mathcal{V} . If $Post(v) \neq \emptyset$, no matter $\theta_{Post(v)}$ is larger than

$\theta_{Post(u_2)}$ or not, according to Lemma 2.5, it is easy to construct other partitions better than the original \mathcal{U} and \mathcal{V} , since the compound groups in these original clusters ($\mathcal{U} = (Pre(u_2) \cup Post(u_1))$, and $\mathcal{V} = (v \cup Post(v))$) does not satisfy property of consecutive group indices within each cluster.

In conclusion, we show that for general cases, if there is any gap in the partition, then we can construct another partition that is better, which is equivalent to that the optimal partition must satisfy consecutive group indices within each cluster. \square

2.10.5 Proof of Theorem 2.6

Proof. Since $U_i(s, p_q)$ is a strictly increasing function in the interval $[0, s_i^*]$, then (2.32) holds, if and only if the following inequality holds:

$$U_i(s_q^*, p_q) \leq U_i(s_{i \rightarrow q}, p_q), \quad \forall i < q. \quad (2.51)$$

Since $t_{1q} > \dots > t_{Kq}$, (2.51) can be simplified to

$$t_{q-1q}^2 \ln t_{q-1q} - (t_{q-1q}^2 - 1) + \frac{\sum_{k=1}^K N_k t_{kq}}{\sum_{k=1}^K N_k + S} (t_{q-1q} - 1) \geq 0, \quad (2.52)$$

where $t_{iq} = \sqrt{\frac{\theta_i}{\theta_q}}$. With a slight abuse of notation, we abbreviate t_{q-1q} as t_q , ($q = 2, \dots, K$) in the sequel. It is easy to see that the following inequality is the necessary and sufficient condition of (2.52) for $q = 2$, and sufficient condition of (2.52) for $q > 2$:

$$t_q^2 \ln t_q - (t_q^2 - 1) + \frac{t_q \sum_{k=1}^{q-1} N_k + N_q}{\sum_{k=1}^K N_k + S} (t_q - 1) \geq 0. \quad (2.53)$$

Let $g(t)$ be the left hand side of the inequality (2.53). It is easy to check that $g(t)$ is a convex function, with $g(1) = 0$, $g(\infty) = \infty$ and $g'(1) < 0$. So there exists a root $t_q > 1$. When $t > t_q$, the inequality (2.53) holds, thus (2.51) holds, and the conclusion in Theorem 2.6 follows. \square

Chapter 3

Profit Maximization of Cognitive Mobile Virtual Network Operator in A Dynamic Wireless Network

In this chapter, we study the profit maximization problem of a cognitive virtual network operator in a dynamic network environment. We consider a downlink OFDM communication system with various network dynamics, including dynamic user demands, uncertain sensing spectrum resources, dynamic spectrum prices, and time-varying channel conditions. In addition, heterogenous users and imperfect sensing technology are incorporated to make the network model more realistic. By exploring the special structural of the problem, we develop a low-complexity on-line control policies that determine pricing and resource scheduling without knowing the statistics of dynamic network parameters. We show that the proposed algorithms can achieve arbitrarily close to the optimal profit with a proper trade-off with the queuing delay.

3.1 Dynamic Spectrum Access

The limited wireless spectrum is becoming a bottleneck for meeting today's fast growing demands for wireless data services. More specifically, there is very little spectrum left that can be licensed to new wireless services and applications. However, extensive field measurements [44] showed that much of the licensed spectrum remains idle most of the time, even in densely populated metropolitan areas such as New York City and Chicago. A potential way to solve this dilemma is to manage and utilize the licensed spectrum resource in a more efficient way.

This is why the concept of *Dynamic Spectrum Access* (DSA) has received enthusiastic support from governments and industries worldwide [3, 45, 46]. We can roughly classify various DSA approaches into two main categories: the *spectrum sensing* based ones and the *spectrum leasing* (or market) based ones. The first category indicates a hierarchical access model, where unlicensed secondary users opportunistically access the under-utilized part of the licensed spectrum, with controlled interference to the licensed primary users. During this process, spectrum sensing helps the secondary users to detect the currently available spectrum resource. In contrast, the second category relates to a dynamic exclusive use model, which allows licensees to trade spectrum usage right to the secondary users. In both categories, it is possible to have a secondary operator coordinating the transmissions of multiple secondary users.

There are pros and cons for both DSA categories. Spectrum sensing detects and identifies the available unused licensed spectrum through technologies such as beacons, geolocation system, and cognitive radio. From the secondary operator's perspective, the spectrum acquired by sensing is *an unreliable resource*, since it cannot determine how much resource is available before sensing. Furthermore,

imperfect sensing may lead to collisions with primary users, and thus reduce the incentives for the licensee to share the spectrum. Therefore the secondary operator needs to carefully design sensing and access algorithm to *control the collision probability under an acceptable level*. In dynamic spectrum leasing, a secondary operator acquires the exclusive right to use spectrum within a limited time period by paying the corresponding leasing price. Thus the spectrum acquired by spectrum leasing is a reliable resource. However, the cost can be high compared to the spectrum sensing cost, and is dynamically changing according to the demand and supply relationship in the market.

In this paper, we will consider a hybrid model, where a secondary operator obtains resources from the primary licensees through both *spectrum sensing* and *dynamic spectrum leasing*, and provides services to the secondary unlicensed users. Our study is motivated by [50, 51], in which the authors introduced the new concept of Cognitive Mobile Virtual Network Operator (C-MVNO). The C-MVNO is a generalization of the existing business model of MVNO [52], which refers to the network operator who does not own a licensed frequency spectrum or even wireless infrastructure, but resells wireless services under its own brand name. The MVNO business model has been very successful after more than 10 years' development, and there are more than 600 MVNOs today [53, 54]. The C-MVNO model generalizes the MVNO model with DSA technologies, which allow the virtual operator to obtain spectrum resources through both spectrum sensing and leasing. The C-MVNO model can be applied to a wide range of wireless scenarios. One example is the IEEE 802.22 standard [56], which suggests that the cognitive radio network using white space in TV spectrum will operate on a point to multipoint basis (*i.e.*, a base station to customer-premises equipments). Such a secondary base station can be operated by a C-MVNO.

The key difference between our work and the ones in [50, 51] is that we study a much more realistic dynamic network in this paper. In [50, 51], the authors formulated the problem based on a static network scenario, and provided interesting equilibrium results through a one-shot Stackelberg game. However, the real network is highly dynamic. For example, users arrive and leave the systems randomly, the statistics of spectrum availability changes over time, and the spectrum-sensing results are imperfect. Also the leasing price is often unpredictable and changing from time to time. These dynamics and realistic concerns make the network model and the corresponding analysis rather challenging.

In this paper, we focus on the profit maximization problem for C-MVNO in a dynamic network scenario. Our key results and contributions are summarized as follows.

- *A dynamic network decision model:* Our model incorporates various key dynamic aspects of a cognitive radio network and the dynamic decision process of a C-MVNO. We model sensing channel availability, leasing market price, and channel conditions as exogenous stochastic processes.
- *Dynamic user demands:* We allow users to dynamically join the network with random demands (file sizes). The demand is affected by both the transmission prices (decision variables) and market states (exogenous stochastic).
- *Realistic cognitive radio model:* We incorporate various practical issues such as imperfect spectrum sensing, primary users' collision tolerance, and sensing technology selection. The operator needs to choose a sensing technology to trade-off between cost and performance.
- *A low-complexity on-line control policy:* By exploiting the special structure of the problem, we design a low-complexity on-line pricing and resource

allocation policy, which can achieve arbitrarily close to the operator's optimal profit. The policy does not require precise information of the dynamic network parameters, has a low system overhead, and is easy to implement.

The remainder of the chapter is organized as follows. In Section II, we introduce the related work. In Section 3.3, we introduce the system model. Section 3.4 describes the problem formulation. In Section 3.5, we propose the profit maximization control (PMC) policy for homogeneous users and analyze its performance. We further extend profit maximization control policy (M-PMC policy) to heterogeneous users in Section 3.6. Section 3.7 provides simulation results for both PMC and M-PMC policies. Finally, we conclude the chapter in Section 3.8.

3.2 Related Work

We can classify the dynamic spectrum access literature into two categories: user-oriented and operator-oriented.

In the user-oriented category, secondary end-users equipped with cognitive radio technologies are the decision-makers in the transmission services. Literature in this category can be further divided into two sub-categories: individual perspective and networking perspective. The individual perspective research adopts a local view of a particular individual user, where resource-consuming spectrum sensing is the central research issue. Many sophisticated algorithms are proposed from an individual secondary user's perspective to achieve a good sensing efficiency [57, 58] or desired tradeoffs between sensing cost and throughput [59–63]. In contrast, the networking perspective research focuses on the interaction among multiple secondary users. The analysis is often based on network optimization [64–66] and game theory [67–69].

In the operator-oriented category, network operators serve as controllers of the networks and players in spectrum market. The literature in this category only started to emerge recently, e.g., [49–51, 70–78]. We can further classify these studies into two clusters: monopoly models with one operator, and oligopoly models with multiple operators.

References [50, 51, 70, 71] studied monopoly models using the Stackelberg game formulation. Daoud et al. in [70] proposed a profit-maximizing pricing strategy for uplink power control problem in wide-band cognitive radio networks. Yu et al. in [71] proposed a pricing scheme that can guarantee a fair and efficient power allocation among the secondary users.

References [49, 72–78] looked at the oligopoly issues, either between two operators [49, 72] or among many operators [73–78]. For the case of two operators, Jia and Zhang in [49] proposed a non-cooperative two-stage game model to study the duopoly competition. Duan et al. in [72] formulated the economic interaction among the spectrum owner, two secondary operators and the users as a three-stage game. For the case of many operators, Ileri et al. in [73] developed a non-cooperative game to model competition of operators in a mixed commons/property-rights regime under the regulation of a spectrum policy server. Elias and Martignon in [75] showed that polynomial pricing functions lead to unique and efficient Nash equilibrium for the two-stage Stackelberg game between network operators and secondary users. Niyato et al. in [74] formulated an evolutionary game for modeling the dynamics of a multiple-seller, multiple-buyer spectrum trading market. In addition, several auction mechanisms were proposed to study the investment problems of cognitive network operators (*e.g.*, [76–78]).

All results mentioned above considered a rather static network model. In contrast, our work adopts a dynamic network model to characterize the stochastic

nature of wireless networks. We will focus on a monopoly model in this chapter.

In this chapter, we use the technique of Lyapunov stochastic optimization to show the optimal performance and the stability of the proposed profit maximizing control algorithm. Several closely related previous results applying Lyapunov stochastic optimization to wireless networks include [65,66,81]. Huang and Neely in [81] considered revenue maximization problem for a conventional wireless access point without considering the cognitive radio technologies. Urgaonkar and Neely in [65] and Lotfinezhad et al. in [66] studied cognitive radio networks based on a user-oriented approach, by designing joint scheduling and resource allocation algorithms to maximize the utility of a group of secondary users. Our paper focused on an operator-oriented approach to address profit maximization problem. In particular, we need to deal with the combinatorial problem of channel selection and channel assignment that usually leads to a high computational complexity. By discovering and utilizing the special problem structure, we design a low-complexity algorithm that is suitable for online implementation.

3.3 System Model

Consider a C-MVNO that provides wireless communications services to its own secondary users by acquiring spectrum resource from some spectrum owner. For example, Google may acquire spectrum from AT&T to provide its own wireless services through the C-MVNO model. The spectrum owner's spectrum can be divided into two types: the *sensing band* and the *leasing band*. In the sensing band, AT&T serves its own primary users, but allows Google to identify available spectrum in this band through spectrum sensing without explicit communications with AT&T. In the leasing band, AT&T will does not allow spectrum sensing, and

will lease the band to Google for economic returns.

More specifically, we consider a time-slotted OFDM system, where the C-MVNO serves the downlink transmissions from its base station to the secondary users. The system model is illustrated in Fig. 3.1. Secondary users randomly arrive at the secondary network and request files with random sizes to be downloaded from the base station. This requested files are queued at the server in the base station until they are successfully transmitted to the requesting users.

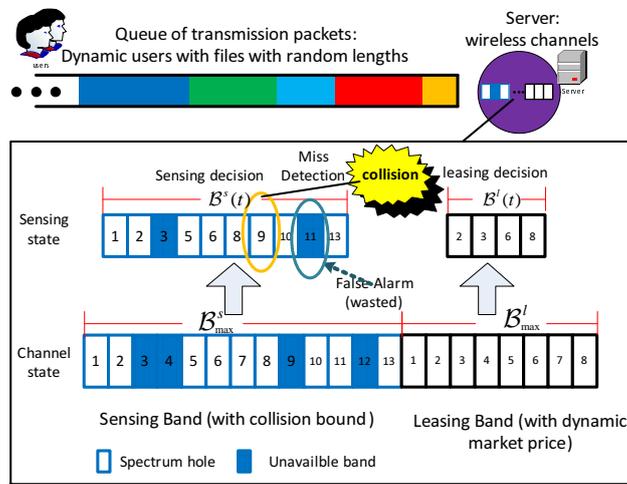


Figure 3.1: Business model of the operator (Cognitive Virtual Network Operator).

Next, we will introduce each part of the system model in more details. The C-MVNO (or “operator” for simplicity) obtains wireless channels through spectrum sensing (Section 3.3.1 and Section 3.3.2) and spectrum leasing (Section 3.3.3), and allocates power over the obtained channels (Section 3.3.4). Secondary users dynamically arrive and request file downloading services (based on the demand model in Section 3.3.5), and we model the requests as a queue (Section 3.3.6).

3.3.1 Imperfect Spectrum Sensing

Sensing band $\mathcal{B}_{\max}^s \triangleq \{1, \dots, B_{\max}^s\}$ includes all channels that the spectrum owner allows sensing by the operator.¹ We define **the state of a channel** $i \in \mathcal{B}_{\max}^s$ **in time slot** t as $S_i(t)$, which equals 0 if channel i is busy (being used by a primary user), and equals 1 if channel i is idle.

We assume that $S_i(t)$ is an i.i.d. Bernoulli random variable, with an idle probability $p_0 \in (0, 1)$ and a busy probability $1 - p_0$. This approximates the reality well if the time slots for secondary transmissions are sufficiently long or the primary transmissions are highly bursty [85]. (We will further study the general Markovian model in Section 3.5.5.) We define the **sensing state of a channel** $i \in \mathcal{B}_{\max}^s$ **in time slot** t as $W_i(t)$, which equals to 0 if channel i is *sensed* busy, and 1 if sensed idle.

Notice that $W_i(t)$ may not equal to $S_i(t)$ due to imperfect sensing. The accuracy of spectrum sensing depends on the sensing technology [86]. If we denote C^s as the sensing cost (per channel)², then we can write the false alarm probability as $P_{fa}(C^s) \triangleq Pr\{W_i = 0 | S_i = 1\}$ (same for all channel i) and the missed detection probability as $P_{md}(C^s) \triangleq Pr\{W_i = 1 | S_i = 0\}$ (same for all channel i). Both functions are decreasing in C^s . Intuitively, a better technology will have a higher cost C^s , a lower false alarm probability $P_{fa}(C^s)$, and a lower missed detection probability $P_{md}(C^s)$. We denote all choices of cost C^s (and thus the corresponding sensing technologies) by a finite set \mathcal{C}^s .

¹The operator will collect the sensing information from a sensor network or geolocation database and provide it to its users, *i.e.*, providing “sensing as service” [83, 84]. This means that the network can accommodate legacy mobile devices without cognitive radio capabilities. For more detailed discussions, see [51].

²The cost corresponds to, for example, power or time used for sensing.

As different channels have different conditions (to be explained in details in Section 3.3.4), the operator needs to decide which channels to sense at the beginning of each time slot. We use $\mathcal{B}^s(t)$ to denote **the set of channels sensed by the operator at time t** , which satisfies

$$\mathcal{B}^s(t) \subseteq \mathcal{B}_{\max}^s, \forall t. \quad (3.1)$$

3.3.2 Collision Constraint

Missed detections in spectrum sensing lead to transmission collisions with the primary users. We denote the **collision in channel $i \in \mathcal{B}_{\max}^s$ at time t** as a binary random variable $X_i(t) \in \{0, 1\}$. We have $X_i(t) \triangleq (1 - S_i(t))W_i(t)$, *i.e.*, the collision happens if and only if the channel is busy but is sensed idle.

To protect primary users' transmissions, the operator needs to ensure that the average collision in each channel i does not exceed a tolerable level η_i (measured in terms of the average number of collisions per unit time) specified by the spectrum owner. The tolerable level η_i can be channel specific, since the primary users in different channels may have different QoS requirements. We define the time-average number of collision in channel i as $\bar{X}_i \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[X_i(\tau)]$. The collision constraints are

$$\bar{X}_i \leq \eta_i, \forall i \in \mathcal{B}_{\max}^s(t). \quad (3.2)$$

3.3.3 Spectrum Leasing with Dynamic Market Price

A spectrum owner may have some channels that do not want to be sensed, for either privacy reasons or the fear of collisions due to sensing errors. However, these channels may not be always fully utilized. The spectrum owner can lease the unused part of these channels to the operator dynamically over time to earn

more revenue. Recall that we denote the set of these channels as the leasing band $\mathcal{B}_{\max}^l \triangleq \{1, \dots, B_{\max}^l\}$. (In general, we may represent it as $\mathcal{B}_{\max}^l(t)$, since our model allows leasing band to be time-varying. For the simplicity of notations, we denote it as \mathcal{B}_{\max}^l whenever it is clear.) We use $\mathcal{B}_i^l(t)$ to denote **the set of channels leased by the operator at time t** , which satisfies

$$\mathcal{B}^l(t) \subseteq \mathcal{B}_{\max}^l, \forall t. \quad (3.3)$$

These channels will be exclusively used by the operator in the current time slot. We denote the leasing price per channel as $C^l(t)$, which stochastically changes according to the supply and demand relationship in the spectrum market (which might involve many spectrum owners and operators). It can be modeled by an exogenous (not affected by this particular operator's decisions) random process with countable discrete states and stationary distribution (not necessarily known by the operator).

3.3.4 Power Allocation

In wireless network, there are usually channel fading due to multipath propagation or shadowing from obstacles. To combat channel fading, it is necessary for the operator to do proper power allocation in both sensing channels and leasing channels to achieve satisfactory data rates. For each channel $i \in \mathcal{B}_{\max} \triangleq \mathcal{B}_{\max}^s \cup \mathcal{B}_{\max}^l$, $h_i(t)$ represents its channel gain in time slot t and follows an i.i.d. distribution over time. Different channels have independent and possibly different channel gain distributions. We assume that secondary users are homogeneous and experience the same channel condition for the same channel. But channel conditions can be different

in different channels.³ (The heterogenous user scenario will be further discussed in Section 3.6.) The operator can measure $h_i(t)$ for each i at the beginning of each slot t , but may not know the distributions. Let $P_i(t)$ denote the **power allocated to channel i at time t** . Since we consider a downlink case here, the operator needs to satisfy the **total power constraint** P_{\max} at its base station,

$$\sum_{i \in \mathcal{B}_{\max}} P_i(t) \leq P_{\max}, \forall t. \quad (3.4)$$

In addition, for a channel $i \in \mathcal{B}_{\max}^s$ in the sensing band, we use the binary variable $I_i(t) \triangleq S_i(t)W_i(t)$ to denote the **transmission result** of a secondary user, *i.e.*, $I_i(t) = 1$ if successful (*i.e.*, $S_i(t) = 1$ and $W_i(t) = 1$) and $I_i(t) = 0$ otherwise (either not sensed, or sensed busy, or sensed idle but actually busy). Based on the discussion of the leasing agreement, we have $I_i(t) \equiv 1$, $i \in \mathcal{B}_{\max}^l$ for all channel in leasing band. Then the rate in channel i at time slot t is (based on the Shannon formula)

$$r_i(t) = I_i(t) \log_2(1 + h_i(t)P_i(t)), \quad (3.5)$$

and **total transmission rate** obtained by the operator is

$$r(t) = \sum_{i \in \mathcal{B}^s(t) \cap \mathcal{B}^l(t)} r_i(t). \quad (3.6)$$

Furthermore, we assume that the operator has a finite maximum transmission rate, *i.e.*, $r(t) \leq r_{\max}$, $\forall t$, under any feasible power allocation.

3.3.5 Demand Model

We will focus on elastic data traffic in this paper. Secondary users randomly arrive at the network to request files with random and finite file sizes (measured in

³This is the case where the users are located close by, and thus the downlink channel condition from the base station to the users is user independent.

the number of packets) from the operator. A user will leave the network once it has downloaded the complete requested file. The operator can price the packet transmission dynamically over time, which will affect the users' arrival rate. For example, a higher price at peak time can refrain users from downloading files, as they can wait until a later time with a lower price. To model this, we use $M(t)$ to denote the random **market state**, which can be measured precisely at the beginning of each time slot t and can help estimate the users demand⁴. The random variable is drawn from a finite set \mathcal{M} over time in an i.i.d. fashion. The distribution of $M(t)$ may not be known by the operator.

At a time t , the operator can decide whether to accept new file downloading requests from newly arrived secondary users. We define the **binary demand control variable** as $O(t)$, where $O(t) = 1$ means that the operator accepts the incoming requests in time t , and $O(t) = 0$ otherwise. When the operator decides to accept new requests of packet transmissions, it will announce **a price $q(t)$ for transmitting one packet (to any user)**. This price will affect the users' incentives of downloading requests, *e.g.*, when price $q(t)$ is high, some users may choose to postpone their requests. Thus the current price directly affects the number of incoming users in the current slot. To model this, we denote the number of incoming users at time t as a discrete random variable $N(t) \triangleq N(M(t), q(t)) \in \mathbb{N} \triangleq \{0, 1, 2, \dots\}$, the distribution of which is a function of the transmission price $q(t)$ and market state $M(t)$. Further, a user n 's requested file size is denoted $L_n(t)$, with $n \in \{1, 2, \dots, N(q(t))\}$, which is assumed to be independent of each other and does not depend on $q(t)$ or $M(t)$. Moreover, we assume that users are using a set $\mathcal{K} = \{1, 2, \dots, K\}$ of different applications, and denote θ_k as the probability that

⁴For example, $M(t)$ can be users' willingness to pay, or whether the system is in peak time or off-peak time.

an incoming user is using application $k \in \mathcal{K}$ with $\sum_{k=1}^K \theta_k = 1$. The distributions of the file length for different applications can be different, and we denote l_k as the expected file length of application $k \in \mathcal{K}$.

To summarize, **users' instantaneous demand at time t** is

$$A(t) \triangleq \sum_{n=1}^{N(M(t), q(t))} L_n(t), \quad (3.7)$$

which is a random variable due to random file sizes and the random number of incoming users (even given $q(t)$ and $M(t)$). We define the **users' (expected) demand function** as $D(t) \triangleq D(M(t), q(t)) \triangleq \mathbb{E}[A(M(t), q(t))]$, and its value is completely determined by $M(t)$ and $q(t)$. We can calculate that $D(M(t), q(t)) = \mathbb{E}[N(M(t), q(t))] \sum_{k \in \mathcal{K}} \theta_k l_k$. Then it is reasonable to assume that the operator can rather accurately characterize the expected number of incoming users $\mathbb{E}[N(M(t), q(t))]$ through long-term observations. Thus the demand function $D(M(t), q(t))$ is known by the operator. We further assume that the instantaneous demand is upper-bounded as $A(t) \leq A_{\max}$ for all t , and that the demand function $D(t)$ is non-negative and non-increasing function of the price $q(t)$. When the price is higher than some upper-bound, *i.e.*, $q(t) \geq q_{\max}$, the demand function $D(t)$ will be zero.

3.3.6 Queuing dynamics

Since we focus on the profit maximization problem in this paper, we will take a simple view of the network and model users' dynamic arrivals and departures as a single server queue. When a user accesses the network, the corresponding file will be queued in a server at the base station, waiting to be transmitted to the user according to the First Come First Serve (FCFS) discipline. Shama and Lin in [87] showed that the single server queue model is a good approximation for an OFDM

system, especially when the number of users and channels are large.

We denote the **queue length** (*i.e.*, the backlog, or the number of all packets from all queued files) at time t as $Q(t)$. Thus the queuing dynamic can be written as

$$Q(t+1) = (Q(t) - r(t))^+ + O(t)A(t), \quad (3.8)$$

where $(a)^+ \triangleq \max(a, 0)$, $r(t)$ and $A(t)$ are the transmission rate and incoming rate at time t , and $O(t)$ is the binary demand control variable (*i.e.*, $O(t) = 1$ means the operator admit the users' transmission requests at time t). Throughout the paper, we adopt the following notion of *queue stability*:

$$\bar{Q} \triangleq \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[Q(\tau)] < \infty. \quad (3.9)$$

3.4 Problem Formulation

For notation convenience, we introduce several condensed notations and use them together with the original notations.

We define $\phi(t) \triangleq (M(t), \mathbf{h}(t), C^l(t))$ as **observable parameters**, including the market state $M(t)$, channel conditions (vector) $\mathbf{h}(t)$, and the leasing price $C^l(t)$ in the spectrum market. Based on previous assumptions, $\phi(t)$'s are i.i.d over time and take values from a finite set Φ .

We define $\gamma(t) \triangleq (O(t), q(t), C^s(t), \mathcal{B}^s(t), \mathcal{B}^l(t), \mathbf{P}(t))$ as **decision variables**, including the demand control variable $O(t)$, the transmission price for users $q(t)$, the sensing cost (with the corresponding sensing technology) $C^s(t)$, the set of sensing channels $\mathcal{B}^s(t)$, the set of leasing channels $\mathcal{B}^l(t)$, and power allocations (vector) $\mathbf{P}(t)$ of the operator. We assume that $\gamma(t)$ takes values form a countable (finite or infinite) set $\Gamma_\phi(t)$, which is a Cartesian product of the feasible regions of

all variables, *i.e.*, non-negative values satisfying constraints (3.1), (3.3), and (3.4). With the condensed notations, functions in this paper can be simply represented as functions of $\gamma(t)$ with parameter $\phi(t)$.

We further define the **instantaneous profit in time t**

$$\begin{aligned} R(t) &\triangleq R(\gamma(t); \phi(t)) \\ &\triangleq q(t)O(t)A(t) - C^s(t)|\mathcal{B}^s(t)| - C^l(t)|\mathcal{B}^l(t)|. \end{aligned} \quad (3.10)$$

The **time average profit** is denoted as

$$\bar{R} \triangleq \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[R(\tau)].$$

All expectations in this paper are taken with respect to system parameters $\phi(t)$ unless stated otherwise.

We look at the profit maximization problem through pricing determination and resource allocations. At the beginning of each time slot t , the operator observes the value of $\phi(t)$ and makes a decision $\gamma(t)$ to maximize the time average profit, subject to the system stability constraint (3.11) and the collision upper-bound requirement (3.12). The Profit Maximization (PM) problem is formulated as

$$\begin{aligned} \mathbf{PM:} \quad &\text{Maximize } \bar{R} \\ &\text{Subject to } \bar{Q} < \infty, \end{aligned} \quad (3.11)$$

$$\bar{X}_i \leq \eta_i, i \in \mathcal{B}_{max}^s, \quad (3.12)$$

$$\text{Variables } \gamma(t) \in \Gamma_{\phi(t)}, \forall t,$$

$$\text{Parameters } \phi(t), \forall t.$$

We represent its optimal solution as $\gamma^*(t) = (O^*(t), q^*(t), C^{s*}(t), \mathcal{B}^{s*}(t), \mathcal{B}^{l*}(t), \mathbf{P}^*(t))$, and denote \bar{R}^* as the maximum profit. The PM problem is an infinite horizon

stochastic optimization problem, which is in general hard to solve directly, especially when the distribution of dynamic parameter $\phi(t)$ is unknown. For example, the future leasing price is hard to predict due to the dynamic supplies and demands in the market; and the primary users' activities can not be estimated precisely before hand.

3.5 Profit Maximization Control (PMC) Policy

Now we adopt Lyapunov stochastic optimization technique to solve the PM problem.

3.5.1 Lyapunov stochastic optimization

We first introduce a virtual queue for constraint (3.12), and then derive the optimal control policy to solve the PM problem through the technique of drift-plus-penalty function minimization [21].

We denote $Z_i(t)$ as the number of collisions happening in sensing channel $i \in \mathcal{B}_{\max}^s$. The counter $Z_i(t)$ can be understood as a “virtual queue”, in which the incoming rate is $X_i(t)$, and the serving rate is η_i (the collision tolerant level). The queue dynamic is

$$Z_i(t+1) = (Z_i(t) - \eta_i)^+ + X_i(t), \quad (3.13)$$

with $Z_i(0) = 0$. By this notion, if the virtual queue is stable, then it implies that the average incoming rate is no larger than the average serving rate. This is just the same as the collision upper-bound constraint (3.12).

We introduce the general queue length vector $\Theta(t) \triangleq \{Q(t), \mathbf{Z}(t)\}$. We then

define the Lyapunov function

$$L(\Theta(t)) \triangleq \frac{1}{2} \left[Q(t)^2 + \sum_{i \in \mathcal{B}_{\max}^s} Z_i(t)^2 \right],$$

and the Lyapunov drift

$$\Delta(\Theta(t)) \triangleq \mathbb{E} [L(\Theta(t+1)) - L(\Theta(t)) | \Theta(t)]. \quad (3.14)$$

According to the Lyapunov stochastic optimization technique, we can obtain instantaneous control policy that can solve the PM problem though minimizing some upper bound of the following drift-plus-penalty function in every slot t :

$$\Delta(\Theta(t)) - V\mathbb{E} [R(t) | \Theta(t)]. \quad (3.15)$$

There are two terms in the above function. The first term is the Lyapunov drift defined in (3.14). It is shown by Lyapunov stochastic optimization [21] that we can achieve the system stabilities (*i.e.*, constraints (3.11) and (3.12) of the PM problem) by showing the existence of a constant upper bound for the drift function. The second term in (3.15) is just the objective of the PM problem, *i.e.*, to minimize the minus profit, which is equivalent to maximize the profit. Here parameter V is introduced to achieve the desired tradeoff between profit and queuing delay in the control policy. We first find an upper bound for (3.15).

By the queue dynamic (3.8), we have

$$\begin{aligned} Q(t+1)^2 &\leq (Q(t) - r(t))^2 + A(t)^2 + 2Q(t)O(t)A(t) \\ &= Q(t)^2 + r(t)^2 + A(t)^2 + 2Q(t)(O(t)A(t) - r(t)). \end{aligned} \quad (3.16)$$

Similarly, for virtual queue (3.13), we have

$$Z_i(t+1)^2 \leq Z_i(t)^2 + \eta_i^2 + X_i(t)^2 + 2Z_i(t)(X_i(t) - \eta_i). \quad (3.17)$$

Substituting (3.16) and (3.17) into (3.15), we have

$$\begin{aligned}
 \Delta(\Theta(t)) - V\mathbb{E}[R(t)|\Theta(t)] &\leq D - V\mathbb{E}[R(t)|\Theta(t)] \\
 &+ Q(t)\mathbb{E}[O(t)A(t) - r(t)|\Theta(t)] \\
 &+ \sum_{i \in \mathcal{B}_{\max}^s} Z_i(t)\mathbb{E}[X_i(t) - \eta_i|\Theta(t)]
 \end{aligned} \tag{3.18}$$

where D is a positive constant satisfying the following condition for all t

$$\begin{aligned}
 D \geq \frac{1}{2} \mathbb{E}[r(t)^2 + (O(t)A(t))^2|\Theta(t)] \\
 + \frac{1}{2} \sum_{i \in \mathcal{B}_{\max}^s} \mathbb{E}[X_i(t)^2 + \eta_i^2|\Theta(t)].
 \end{aligned}$$

We further expand the right hand side of (3.18):

$$\begin{aligned}
 \Delta(\Theta(t)) - V\mathbb{E}[R(t)|\Theta(t)] &\leq D - D_1(t) \\
 &- V\mathbb{E}[q(t)O(t)A(t) - C^s(t)|\mathcal{B}^s(t) - C^l(t)|\mathcal{B}^l(t)|\Theta(t)] \\
 &+ Q(t)\mathbb{E}[O(t)A(t) - r(t)|\Theta(t)] \\
 &+ \sum_{i \in \mathcal{B}_{\max}^s} Z_i(t)\mathbb{E}[X_i(t)|\Theta(t)]
 \end{aligned}$$

where $D_1(t) \triangleq \sum_{i \in \mathcal{B}_{\max}^s} Z_i(t)\eta_i$ is a known constant at time t , since the values of $Z_i(t)$ s are known at time t .

Note that the collision between secondary and primary users can only happen in the channels that have been chosen for sensing, *i.e.*, $i \in \mathcal{B}^s(t)$. By this fact and

(3.6), we have

$$\begin{aligned}
 & \Delta(\Theta(t)) - V\mathbb{E}[R(t)|\Theta(t)] \leq D - D_1(t) \\
 & - V\mathbb{E} \left[q(t)O(t)A(t) - \sum_{i \in \mathcal{B}^s(t)} C^s(t) - \sum_{i \in \mathcal{B}^l(t)} C^l(t) | \Theta(t) \right] \\
 & + Q(t)\mathbb{E} \left[O(t)A(t) - \sum_{i \in \mathcal{B}^s(t) \cup \mathcal{B}^l(t)} r_i(t) | \Theta(t) \right] \\
 & + \sum_{i \in \mathcal{B}^s(t)} Z_i(t)\mathbb{E}[X_i(t)|\Theta(t)]
 \end{aligned}$$

To simplify the above expression, we introduce two new notations: channel selection $\mathcal{B}(t) \triangleq \mathcal{B}^s(t) \cup \mathcal{B}^l(t)$, and channel cost

$$C_i(t) \triangleq \begin{cases} \widetilde{C}^s(t) & \text{if } i \in \mathcal{B}^s(t), \\ C^l(t) & \text{if } i \in \mathcal{B}^l(t), \end{cases} \quad (3.19)$$

where the $\widetilde{C}^s(t)$ is the virtual sensing cost, which is defined as $\widetilde{C}^s(t) \triangleq C^s(t) + (1/V)Z_i(t)\mathbb{E}[X_i(t)|\Theta(t)]$. It shows that the cost of a sensing channel not only depends on the sensing cost, but also on the collision history in this channel. The more frequently the collision happened in this channel before, the higher the cost, which makes the operator more conservative about choosing this sensing channel.

It then follows:

$$\begin{aligned}
 & \Delta(\Theta(t)) - V\mathbb{E}[R(t)|\Theta(t)] \leq D - D_1(t) \\
 & + V\mathbb{E} [((1/V)Q(t) - q(t))O(t)A(t)|\Theta(t)] \\
 & + V\mathbb{E} \left[\sum_{i \in \mathcal{B}(t)} C_i(t) - (1/V)Q(t)r_i(t) | \Theta(t) \right]. \quad (3.20)
 \end{aligned}$$

Choosing the control decision variables to minimize the right hand side of the above inequality (3.20) for each time t leads to a control algorithm, which we call

Profit Maximization Control (PMC) policy that will be discussed in details in Section 3.5.2.

3.5.2 Profit Maximization Control (PMC) policy

It is clear that minimizing the right hand side of (3.20) is equivalent to minimizing the last two terms in (3.20). Note that the last two terms are decoupled in decision variables, thus we have the two parallel parts in the PMC policy as follows:

3.5.2.1 Revenue Maximization

Here we determine two variables: the transmission price $q(t)$ and the market control decision $O(t)$. The optimal transmission price $q(t)$ is obtained by solving the following revenue maximization problem:

$$\begin{aligned} & \text{Maximize } q(t)D(q(t), M(t)) - \frac{Q(t)}{V}D(q(t), M(t)) & (3.21) \\ & \text{Variables } q(t) \geq 0 \end{aligned}$$

To obtain the above problem formulation of revenue maximization, we use the fact that the demand function $D(M(t), q(t)) \triangleq \mathbb{E}[A(t)]$, which is independent of the queuing states of the system.

Note that the first term in (3.21) is just the revenue that the operator collects from its users. The second term can be viewed as a shift of the queuing effect, which is introduced by the Lyapunov drift for system stability.

If the maximum objective in (3.21) (under the optimal choice of $q(t)$) is positive, the operator sets the demand control variable $O(t) = 1$ and accepts the present incoming requests $A(t)$ at the price $q(t)$. Otherwise, the operator sets $O(t) = 0$ and rejects any new requests.

3.5.2.2 Cost Minimization

We determine channels selection $\mathcal{B}(t)$, sensing technology (or cost) $C^s(t)$, and power allocation $P(t)$, by solving the following optimization problem to control the costs of the operator to provide transmission services to its users.

$$\text{Minimize } \sum_{i \in \mathcal{B}(t)} C_i(t) - \frac{Q(t)}{V} \mathbb{E} [r_i(t) | \Theta] \quad (3.22)$$

$$\text{Subject to } (3.1), (3.3), (3.4)$$

$$\text{Variables } C^s(t), \mathcal{B}^s(t), \mathcal{B}^l(t), P_i(t) \geq 0$$

To obtain the above problem formulation of cost minimization, we use the fact that $X_i(t) = (1 - S_i(t))W_i(t)$, which is independent of the queuing state. Thus the virtual sensing cost can be updated as $\widetilde{C}^s(t) = C^s(t) + (1/V)Z_i(t)(1 - p_0)P_{md}(C^s(t))$, which increases with the virtual queue and missed detection probability.

Note that the first term in the summation in (3.22) is the cost of each channel. The second term in the summation is a queuing-weighted expected transmission rate, again is a shift introduced by Lyapunov drift for system stability. This shift can be also viewed as the “gain” collected from the channel to help clear the queue.

3.5.2.3 Intuitions behind the PMC policy

We discuss some intuitions behind the PMC policy. To maximize the profit, the operator needs to perform revenue maximization and cost minimization. To guarantee the queuing stability, some shifts (*i.e.*, all queue-related terms) are introduced by the Lyapunov drift in these problems. In Appendix Sec. 3.9.5, we show that the queueing effect will increase the optimal price announced by the operator (comparing with not considering queueing), as a higher price will reduce the

users' demands and maintain the system stability.

The Lyapunov stochastic optimization approach provides a way to decompose a long-term average goal (*e.g.*, the PM problem) into instantaneous optimization problems (*e.g.*, revenue maximization and cost minimization problems in the PMC policy). In the stochastic optimization problem, the current decisions always have impacts on the future problems. These impacts are characterized and incorporated by the queueing shift terms in the instantaneous optimization problems. Therefore, we can achieve the long term goal through focusing on the instantaneous decisions in every time slot. The flowchart for the PMC policy is illustrated in Fig. 3.2.

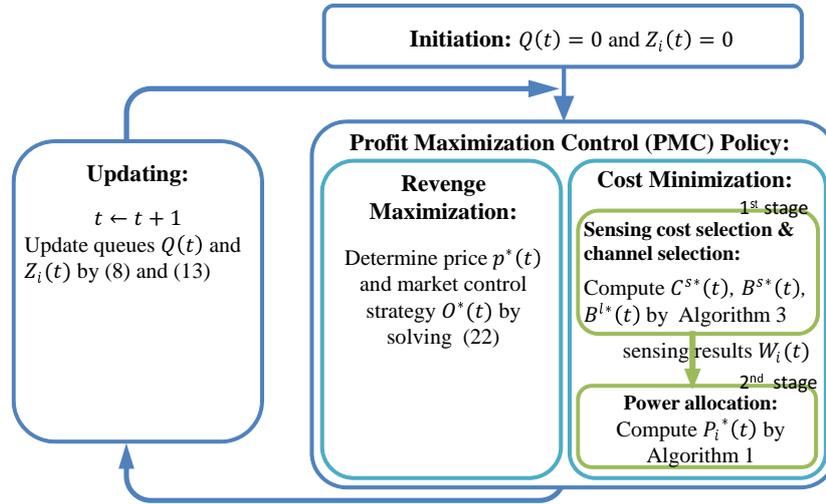


Figure 3.2: Flowchart of the dynamic PMC policy

Although the revenue maximization problem is relatively easy to solve, the cost minimization problem is very complicated. It is actually a two-stage decision problem. In the first stage, the operator determines the sensing technology, and chooses which channels to sense and which channels to lease. Then spectrum sensing is performed to identify available channels. With this information, the operator further allocates downlink transmission power in the available chan-

nels (sensed idle ones and leasing ones). In Section 3.5.3, we focus on designing algorithms to solve the cost minimization problem.

3.5.3 Algorithms for Cost Minimization Problem

Now we use backward induction to solve the cost minimization problem.

3.5.3.1 The Second Stage Problem

We first analyze the power allocation in the second stage, where the sensing results $W_i(t)$, the channel selection $\mathcal{B}^s(t)$, and the sensing technology $C^s(t)$ have been determined. Therefore, the power allocation problem of (3.22) is as follows:

$$\begin{aligned}
 & \text{Maximize} && \sum_{i \in \mathcal{B}(t)} \omega_i(t) \log(1 + h_i(t)P_i(t)) && (3.23) \\
 & \text{Subject to} && \sum_{i \in \mathcal{B}_{\max}} P_i(t) \leq P_{\max}, \\
 & \text{Variables} && P_i(t) \geq 0
 \end{aligned}$$

where

$$\omega_i(t) \triangleq \begin{cases} \omega_0 \triangleq \mathbb{E}[S_i(t)|W_i(t) = 1], & \text{if } i \in \mathcal{B}^s(t), \\ 1, & \text{if } i \in \mathcal{B}^l(t). \end{cases} \quad (3.24)$$

and we can calculate

$$\begin{aligned}
 & \mathbb{E}[S_i(t)|W_i(t) = 1] \\
 &= \frac{p_0(1 - P_{fa}(C^s(t)))}{p_0(1 - P_{fa}(C^s(t))) + (1 - p_0)P_{md}(C^s(t))}. \quad (3.25)
 \end{aligned}$$

By using the Lagrange duality theory, we can show that the problem (3.23) has the following optimal solution

$$P_i^*(t) = \begin{cases} \omega_i(t) \left(\frac{1}{\lambda(t)} - \frac{1}{\omega_i(t)h_i(t)} \right)^+ & i \in \mathcal{B}(t), \\ 0 & i \notin \mathcal{B}(t), \end{cases} \quad (3.26)$$

where $\lambda(t)$ is the Lagrange multiplier of the total power constraint (3.4). The optimal value of $\lambda(t)$ is the following water filling solution,

$$\lambda(t) = \frac{\sum_{i \in \mathcal{B}_p(t)} \omega_i(t)}{P_{\max} + \sum_{i \in \mathcal{B}_p(t)} \frac{1}{h_i(t)}}, \quad (3.27)$$

where $\mathcal{B}_p(t) \triangleq \{i \in \mathcal{B}(t) : P_i(t) > 0\}$. Note that (3.27) is a fixed-point equation of $\lambda(t)$, and the precise value of $\lambda(t)$ is not given here.

When the values of all parameters (*i.e.*, $h_i(t), \omega_i(t)$) are given, we can use a simple water level searching Algorithm 8 (shown in Appendix 3.9.1 and similar as the searching algorithms in [88, 89]) to determine the exact optimal value of $\lambda(t)$. This algorithm involve sorting the channels according to the channel gains, and the complexity of Algorithm 8 is $\mathcal{O}(|\mathcal{B}_{\max}|^2 \log(|\mathcal{B}_{\max}|))$.

3.5.3.2 The First Stage Problem

Let us consider the first stage problem to determine the sensing technology, the sensing set, and the leasing set. Note that since the sensing has not been performed at this stage yet, thus the sensing result $W_i(t)$ is not known. We denote

$$\alpha_i(t) = \begin{cases} \alpha_0 \triangleq \mathbb{E}[S_i(t)W_i(t)] & \text{if } i \in \mathcal{B}^s(t) \\ 1 & \text{if } i \in \mathcal{B}^l(t), \end{cases} \quad (3.28)$$

and we can calculate

$$\mathbb{E}[S_i(t)W_i(t)] = p_0(1 - P_{fa}(C^s(t))) \quad (3.29)$$

Substitute the optimal power allocation (3.26) into the problem (3.22), we have

$$\text{Minimize } \sum_{i \in \mathcal{B}(t)} C_i(t) - \frac{Q(t)}{V} \alpha_i(t) \left(\log \left(\frac{h_i(t) \omega_i(t)}{\lambda(t)} \right) \right)^+ \quad (3.30)$$

$$\text{Subject to } \mathcal{B}^s(t) \subseteq \mathcal{B}_{\max}^s, \mathcal{B}^l(t) \subseteq \mathcal{B}_{\max}^l, C^s(t) \in \mathcal{C}$$

$$\text{Variables } \mathcal{B}^s(t), \mathcal{B}^l(t), C^s(t)$$

We first consider the above problem for a fixed sensing cost $C^s(t)$. This problem is a combinatorial optimization problem of $\mathcal{B}^s(t)$ and $\mathcal{B}^l(t)$. The worst case of searching complexity (*i.e.*, exhaustive searching) can be $\mathcal{O}(2^{|\mathcal{B}_{\max}^s|})$, exponential in the number of total channels.

However, we can reduce the complexity by exploring the special structure of this problem.

Proposition 3.1. (*Threshold Property*)

- We rearrange the leasing channel indices $i \in \mathcal{B}_{\max}^l$ in the decreasing order of $g_i(t)$, which is defined as

$$g_i(t) \triangleq h_i(t) \exp\left(-\frac{C_i(t)}{\frac{Q(t)}{V}}\right). \quad (3.31)$$

There exists a threshold index i_{th}^l , such that a channel i is chosen for leasing (*i.e.*, $i \in \mathcal{B}^l(t)$) if and only if $i \leq i_{th}^l$.

- We rearrange the sensing channel indices $j \in \mathcal{B}_{\max}^s$ in the decreasing order of $g_j(t)$, which is defined as

$$g_j(t) \triangleq \omega_0 h_j(t) \exp\left(-\frac{C_j(t)}{\frac{Q(t)}{V} \alpha_0}\right). \quad (3.32)$$

For all leasing channels $j \in \mathcal{B}_{\max}^s$, there exists a threshold index j_{th}^s , such that a channel j is chosen for sensing (*i.e.*, $j \in \mathcal{B}^s(t)$) if and only if $j \leq j_{th}^s$.

Proof. For each channel in the optimal channel selection set $i \in \mathcal{B}^*(t)$, it satisfies the following condition

$$C_i(t) \leq \frac{Q(t)}{V} \alpha_i(t) \left(\log \left(\frac{\omega_i(t) h_i(t)}{\lambda(t)} \right) \right)^+. \quad (3.33)$$

This result is easy to see from the objective function in (3.30): to optimize the profit, we should only pick the channel with its cost no larger than its gain. Thus

by (3.31) and (3.32), the optimization problem in (3.30) can be written in the following equivalent form:

$$\begin{aligned}
 & \text{Maximize} \quad \sum_{i \in \mathcal{B}^l(t)} \left(\log \left(\frac{g_i(t)}{\lambda(t)} \right) \right)^+ + \alpha_0 \sum_{j \in \mathcal{B}^s(t)} \left(\log \left(\frac{g_j(t)}{\lambda(t)} \right) \right)^+ & (3.34) \\
 & \text{Subject to} \quad \mathcal{B}^s(t) \subseteq \mathcal{B}_{\max}^s, \quad \mathcal{B}^l(t) \subseteq \mathcal{B}_{\max}^l \\
 & \text{Variables} \quad C^s(t), \mathcal{B}^s(t), \mathcal{B}^l(t)
 \end{aligned}$$

Thus by the log function in the objective of (3.34), the threshold property immediately follows. \square

This proposition suggests that we should select the channel with a large g_i (for leasing channels) or g_j (for sensing channels). Note that as defined in (3.31) and (3.32), g_i and g_j are equal to channel information (*i.e.*, h_i for leasing channels, $\omega_j h_j$ for sensing channels) multiplying a decaying factor related to the channel cost. They can be understood as virtual channel gains when the channel costs are taken into consideration. A large value of g_i or g_j means that the channel is cost-effective, *i.e.*, the channel has a good channel gain as well as a low cost.

By Proposition 3.1, it is clear that we can determine the optimal channel selection by an exclusive search of the optimal sensing and leasing thresholds. (A pseudo code of the searching Algorithm 9 is given in Appendix 3.9.2.) Thus the searching complexity is reduced to $\mathcal{O}(|\mathcal{B}_{\max}|^2)$ (or more exactly, $\mathcal{O}(|\mathcal{B}_{\max}^s| \times |\mathcal{B}_{\max}^l|)$), given the channel indices are rearranged as the Proposition 3.1. We can adopt established sorting algorithms [90] to finish the index rearrangement with a complexity $\mathcal{O}(|\mathcal{B}_{\max}| \log(|\mathcal{B}_{\max}|))$. Thus the total complexity of finding the optimal channel selection is $\mathcal{O}(|\mathcal{B}_{\max}|^3 \log(|\mathcal{B}_{\max}|))$.

Note that in real systems, the channel conditions and the leasing cost may not change as frequently as every time slot. We usually can update these network

parameters every time frame (which is composed by several time slots instead of one time slot). Accordingly, the above algorithm will also be operated based on the time frames, which will greatly reduce the computation complexity in practice.

Furthermore, let us find the optimal sensing cost $C^s(t)$ by enumerating all possible sensing costs $C^s(t) \in \mathcal{C}^s$. For the sensing cost $C^s(t)$, we denote the objective value in (3.34) as $U(C^s(t))$ and the optimal channel selection set as $\mathcal{B}(C^s(t))$. The corresponding pseudo code is given in Algorithm 6, the complexity of which is $\mathcal{O}(|\mathcal{C}| \times |\mathcal{B}_{\max}|^3 \log(|\mathcal{B}_{\max}|))$.

Algorithm 6 Optimal Sensing Cost and Channel Selection

- 1: $U^* \leftarrow 0$
 - 2: **for** $C^s(t) \in \mathcal{C}^s$ **do**
 - 3: Determine the optimal channel selection $\mathcal{B}(C^s(t))$ (by the exclusive search for the optimal sensing and leasing thresholds, see Appendix 3.9.2)
 - 4: Calculate $U(C^s(t))$
 - 5: **if** $U^* > U$ **then**
 - 6: $U^* \leftarrow U, C^{s*}(t) \leftarrow C^s(t), \mathcal{B}^*(t) \leftarrow \mathcal{B}(C^s(t))$
 - 7: **end if**
 - 8: **end for**
-

So far, we have completely solved the two-stage optimization problem in (3.22). For each time t , the operator first runs Algorithm 6 to choose the channel sets $\mathcal{B}^*(t) = \mathcal{B}^{s*}(t) \cup \mathcal{B}^{l*}(t)$. Then it uses the sensing technology with a cost $C^{s*}(t)$ to sense channels in $\mathcal{B}^{s*}(t)$. Based on the sensing results, it further runs Algorithm 8 to determine the power allocation $P_i^*(t), i \in \mathcal{B}^*(t)$.

3.5.3.3 Sensing Vs. Leasing

We are also interested in how the PMC policy makes the best tradeoff between sensing and leasing based on the sensing cost $C^s(t)$ and the leasing cost $C^l(t)$. To make the comparison easy to understand, we will consider perfect sensing with no sensing errors (*i.e.*, $\omega_0 = 1$ and $\alpha_0 = p_0$). We will further assume that a leasing channel i and a sensing channel j have the same channel gain $h_i(t) = h_j(t)$. Finally, we assume that two channels have the same availability-price-ratio, *i.e.*, the costs satisfy $C^s(t) = \alpha_0 C^l(t)$. We want to answer the following question: *is the PMC policy indifferent in choosing either of the two channel?*

By (3.31) and (3.32), we have $g_i = g_j$ for these two channels. By (3.34), we can calculate the net gains by channel i and j : $\left(\log\left(\frac{g_i(t)}{\lambda(t)}\right)\right)^+ \geq \alpha_0 \left(\log\left(\frac{g_j(t)}{\lambda(t)}\right)\right)^+$. To maximize the objective in (3.34), it is clear that PMC policy will prefer the leasing channel i over the sensing channel j , and this tendency increases as α_0 decreases. If we view the channel unavailability $(1 - \alpha_0)$ as the risk of choosing the sensing channel, then the PMC policy is a risk averse one. This is mainly due to the concavity of the rate function. This preference order will also hold in the imperfect sensing case, in which case we will have $g_i > g_j$ and $\left(\log\left(\frac{g_i(t)}{\lambda(t)}\right)\right)^+ \geq \alpha_0 \left(\log\left(\frac{g_j(t)}{\lambda(t)}\right)\right)^+$.

3.5.4 Performance of the PMC Policy

We show the performance bounds of the PMC Policy as follows:

Theorem 3.1. *For any positive value V , the PMC Policy has the following properties:*

- (a) *The queue stability (3.11) and collision constraints (3.12) are satisfied. The*

queue length is upper bounded by

$$Q(t) \leq Q_{\max} \triangleq Vq_{\max} + A_{\max}, \quad \forall t; \quad (3.35)$$

and the virtual queue length is upper-bounded by

$$Z_i(t) \leq Z_{\max} \triangleq \kappa(Vq_{\max} + A_{\max}) + 1, \quad \forall i, t. \quad (3.36)$$

where

$$\kappa \triangleq \frac{r_{\max} p_0 (1 - P_{fa}(C^{s0}))}{(1 - p_0) P_{md}(C^{s0})}$$

$$\text{and } C^{s0} \triangleq \max_{C^s} \frac{p_0 (1 - P_{fa}(C^s))}{(1 - p_0) P_{md}(C^s)}.$$

(b) The average profit \overline{R}_{PMC} obtained by the PMC policy satisfies

$$\inf \overline{R}_{PMC} \geq \overline{R}^* - O(1/V), \quad (3.37)$$

where \overline{R}^* is the optimal value of the PM problem.

According to the Little's law, the average queuing delay is proportional to the queue length. Thus users experience bounded queuing delays under the PMC algorithm by (3.35). By (3.37), we find that the profit obtained by the PMC Policy can be made closer to the optimal profit by increasing V . However, as V increases, the queuing delay also increases as shown in (3.35). The best choice of V depends on the desired trade-off between queuing delay and profit optimality.

A detailed proof of Theorem 3.1 is provided in Appendices 3.9.3 and 3.9.4.

3.5.5 Extension: More General Model of Primary Users' Activities

In the previous analysis, we assume that primary users' activities in each sensing channel is a simple i.i.d. Bernoulli random process, and derive the PMC policy for the operator's profit maximization problem. We find that the PMC policy also

holds for the following more general Markov chain model of the primary users' activities shown in Fig. 3.3. In this model, for any time t , $S_i(t)$ is unknown, but the history information $S_i(t - 1)$ is known, and also the transition probabilities $Pr(S_i(t) = s' | S_i(t - 1) = s) \triangleq p_{s \rightarrow s'}^i, s \in \{0, 1\}, s' \in \{0, 1\}, i \in \mathcal{B}_{\max}^s$ are known from long-time statistics.

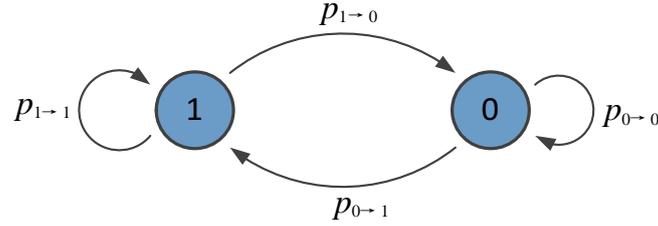


Figure 3.3: Markov chain model of the PUs' activities

All previous analysis for PMC policy will still hold if we update two parameters $\omega_i(t)$ and $\alpha_i(t)$ as follows:

$$\omega_i(t) \triangleq \begin{cases} \mathbb{E}[S_i(t) | W_i(t) = 1, S_i(t - 1)], & \text{if } i \in \mathcal{B}^s(t) \\ 1, & \text{if } i \in \mathcal{B}^l(t) \end{cases}$$

where

$$\begin{aligned} & \mathbb{E}[S_i(t) | W_i(t) = 1, S_i(t - 1) = s] \\ &= \frac{p_{s \rightarrow 1}^i (1 - P_{fa}(C^s(t)))}{p_{s \rightarrow 1}^i (1 - P_{fa}(C^s(t))) + p_{s \rightarrow 0}^i P_{md}(C^s(t))}; \end{aligned}$$

and

$$\alpha_i(t) = \begin{cases} \mathbb{E}[S_i(t) W_i(t) | S_i(t - 1)] & \text{if } i \in \mathcal{B}^s(t) \\ 1 & \text{if } i \in \mathcal{B}^l(t) \end{cases}$$

where

$$\mathbb{E}[S_i(t) W_i(t) | S_i(t - 1) = s] = p_{s \rightarrow 1}^i (1 - P_{fa}(C^s(t))).$$

There is no change in the revenue maximization part, and the cost minimization part still involves a combinatorial optimization problem. But the complexity of solving cost minimization problem becomes $\mathcal{O}(|\mathcal{C}| \times 2^{|\mathcal{B}_{\max}^s|} |\mathcal{B}_{\max}^l| + 1)$, since we lose the structure information in sensing channels, *i.e.*, the threshold structure does not hold for sensing channels. In the worst case ($\mathcal{B}_{\max} = \mathcal{B}_{\max}^s, \mathcal{B}_{\max}^l = \emptyset$), it comes back to $\mathcal{O}(|\mathcal{C}| \times 2^{|\mathcal{B}_{\max}|})$, which is the complexity of the exhaustive search without considering the threshold structure.

Let us further consider a special Markov chain model where the transition probability for each sensing channel is the same, *i.e.*, $Pr(S_i(t) = s' | S_i(t-1) = s) \triangleq p_{s \rightarrow s'}, \forall i \in \mathcal{B}_{\max}^s$. In this model, all sensing channels can be categorized into two types, channels being busy in the last slot (*i.e.*, $S_i(t-1) = 0$), or channels being idle in the last slot (*i.e.*, $S_i(t-1) = 1$). We can still show threshold structures for both types. Thus the complexity is reduced to $\mathcal{O}(|\mathcal{C}| \times |\mathcal{B}_{\max}|^4 \log(|\mathcal{B}_{\max}|))$.

The above analysis shows that it is critical to exploit the problem structure to reduce the algorithm complexity.

3.6 Heterogeneous Users

In Section 3.4, we adopt the single queue analysis for homogeneous users who are assumed to be nearby and have the same channel condition on each channel. However, the single queue analysis can not work for a more general scenario of heterogeneous users, where users can be located at different places, and have different channel conditions. In this section, we introduce the multi-queue model to deal with the heterogenous user scenario as shown in Fig. 3.4.

We divide the total coverage of the secondary base station into $\mathcal{J} \triangleq \{1, 2, \dots, J\}$ disjoint small areas (illustrated as hexagons in Fig. 3.4) according to users' differ-

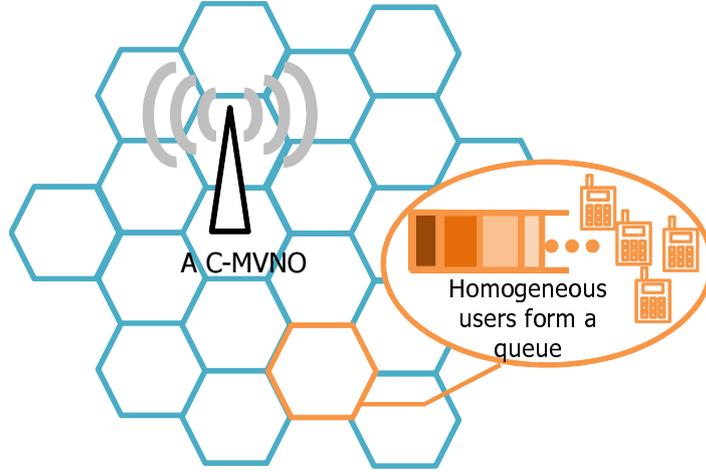


Figure 3.4: Heterogeneous user model: Users in a hexagons are nearby homogeneous users, who have the same channel experience. Users in deferent hexagons can have different channel experience.

ent channel experiences. Users in one of these small area are nearby homogeneous users. They share the same channel conditions, and form a queue based on the FCFS discipline. We use $Q_j(t)$ to denote the queue length in area j . Since the queue and the corresponding area is one-to-one mapping, we also call the users in area j as queue j users.

For each queue $j \in \mathcal{J}$, $h_{ij}(t)$ represents the users' channel gain to channel $i \in \mathcal{B}_{\max}$ at time t , which follows an i.i.d distribution over time. The indicator variable $T_{ij}(t) \in \{0, 1\}$ denotes **the operator's channel assignment at time t** : $T_{ij}(t) = 1$ if channel i is allocated to queue j , and $T_{ij}(t) = 0$ otherwise. Meanwhile, the assignment T_{ij} must satisfy

$$\sum_{j \in \mathcal{J}} T_{ij}(t) \leq 1, \forall t. \quad (3.38)$$

The power allocation for queue j on channel i is denoted by $P_{ij}(t)$. The total

power allocation must satisfy

$$\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{B}_{\max}} P_{ij}(t) \leq P_{\max}, \forall t, \quad (3.39)$$

Thus the rate of queue $j \in \mathcal{J}$ can be calculated

$$r_j(t) = \sum_{i \in \mathcal{B}(t)} I_i(t) T_{ij}(t) \log \left(1 + \frac{h_{ij}(t) P_{ij}(t)}{T_{ij}(t)} \right) \quad (3.40)$$

where $I_i(t)$ is the transmission result in channel i , following the same definition in Section 3.3.4.

For each queue j , we follow the same demand model as in Section 3.3.5 for homogenous users. We assume the number of incoming users, the market state, and the user's instantaneous demand are i.i.d among different queues, and denote them as $N_j(t)$, $M_j(t)$, and $A_j(t)$ respectively. The market control variable and price for queue j are denoted as $O_j(t)$ and $q_j(t)$. The queuing dynamic for queue j is as follows:

$$Q_j(t+1) = (Q_j(t) - r_j(t))^+ + O_j(t) A_j(t). \quad (3.41)$$

Thus the homogeneous user model in Section 3.4 can also be viewed as a special case of the heterogeneous user model, where \mathcal{J} is a singleton.

3.6.1 Multi-queue Profit Maximization Control (M-PMC) Policy

3.6.1.1 Revenue Maximization

For any queue $j \in \mathcal{J}$, we compute the optimal transmission price $q_j^*(t)$ by solving the following problem.

$$\begin{aligned} & \text{Maximize} \quad \left(q_j(t) - \frac{Q_j(t)}{V} \right) D(q_j(t), M_j(t)) \\ & \text{Variables} \quad q_j(t) \geq 0 \end{aligned} \quad (3.42)$$

If the maximum objective in (3.42) is positive, the operator sets transmission control variable $O_j^*(t) = 1$ and accepts users' new file download requests at the price $q_j^*(t)$. Otherwise, the operator sets $O_j^*(t) = 0$ and rejects any new requests.

3.6.1.2 Cost Minimization

We solve the following optimization problem to determine sensing cost and resource allocation:

$$\begin{aligned} \text{Minimize } & \sum_{i \in \mathcal{B}(t)} C_i(t) - \sum_{j \in \mathcal{J}} \frac{Q_j(t)}{V} \mathbb{E}[r_j(t)] & (3.43) \\ \text{Subject to } & (3.1), (3.3), (3.39), (3.38) \\ \text{Variables } & C^s(t), \mathcal{B}^s(t), \mathcal{B}^l(t), P_{ij}(t) \geq 0, T_{ij}(t) \in \{0, 1\} \end{aligned}$$

where the cost $C_i(t)$ follows the same definition of (3.19). Similar to the homogeneous model in Section 3.5.3, it is a two-stage decision problem. In the first stage, we determine the sensing technology, and choose the sensing channels and the leasing channels. Then in the second stage, we determine the channel assignment and power allocation based on the sensing results. We use backward induction to solve this problem (3.43). To simplify the notation, we will ignore the time index in the following analysis.

We first analyze the channel assignment and power allocation in the second stage. In this stage, since the sensing results W_i , the channels \mathcal{B}^s , and the sensing technology C^s are determined, the second stage problem of (3.22) is shown as

follows:

$$\text{Maximize } \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{B}} Q_j \omega_i T_{ij} \log \left(1 + \frac{h_{ij} P_{ij}}{T_{ij}} \right) \quad (3.44)$$

Subject to (3.39), (3.38)

Variables $P_{ij} \geq 0, T_{ij} \in \{0, 1\}$

where ω_i follows the same definition of (3.24).

Compared to the power allocation problem in (3.23), the binary channel assignment variables T_{ij} 's make the second stage problem much more complex.

We first solve problem (3.44) for fixed T_{ij} 's, thus the remaining power allocation problem is a convex optimization problem. Following the same method of solving power allocation for homogeneous users as in (3.26), we have

$$P_{ij} = T_{ij} Q_j \omega_i \left(\frac{1}{\lambda} - \frac{1}{Q_j \omega_i h_{ij}} \right)^+, \quad i \in \mathcal{B}. \quad (3.45)$$

where λ is the Lagrange multiplier of the total power constraint (3.39), which satisfies

$$\lambda = \frac{\sum_{i \in \mathcal{B}_p} \omega_i Q_j T_{ij}}{P_{\max} + \sum_{i \in \mathcal{B}_p} \frac{T_{ij}}{h_i(t)}}, \quad (3.46)$$

where $\mathcal{B}_p \triangleq \{i \in \mathcal{B} : P_{ij} > 0\}$. When T_{ij} is known, we can design a simple search algorithm similar to Alg. 8 to determine the optimal value of λ .

We then substitute this result in (3.44), and further maximize the objective over T_{ij} 's. Then we have

$$\text{Maximize } \sum_{i \in \mathcal{B}} \omega_i \sum_{j \in \mathcal{J}} Q_j T_{ij} \left(\log \left(\frac{\omega_i Q_i h_i}{\lambda} \right) \right)^+ \quad (3.47)$$

Subject to $\sum_{j \in \mathcal{J}} T_{ij} \leq 1$

Variables $T_{ij} \in \{0, 1\}$

For each channel $i \in \mathcal{B}$, let us denote the set

$$\mathcal{J}_i^* \triangleq \arg \max_{j \in \mathcal{J}} Q_j \left(\log \left(\frac{\omega_i Q_j h_{ij}}{\lambda} \right) \right)^+ \quad (3.48)$$

Here the solution \mathcal{J}_i^* is a set of indices of the chosen queues. If \mathcal{J}_i^* is a singleton, then we denote its unique element as j_i^* . If \mathcal{J}_i^* is not singleton, since all elements in \mathcal{J}_i^* lead to the same value in objective, then we can randomly pick one of the its element and denote it as j_i^* .

Since the objective in problem (3.47) is linear in T_{ij} , it is easy to see the optimal solution is

$$T_{ij} = \begin{cases} 1, & \text{if } j = j_i^*, i \in \mathcal{B} \\ 0, & \text{otherwise.} \end{cases} \quad (3.49)$$

Now let us consider how to calculate the value of λ and j_i^* . By (3.46) and (3.48), we find that they are actually coupled together. To determine λ in (3.46), we need to know T_{ij} (or equivalently \mathcal{J}_i^* , j_i^*), *i.e.*, which queue is chosen for which channel. But determining \mathcal{J}_i^* in (3.48) requires the value of λ . One way to solve this problem is to enumerate every possible channel assignment combinations to find the solutions satisfying both (3.46) and (3.48). Since each channel $i \in \mathcal{B}_{\max}$ can be assigned to one of J queues, there are a total of $|\mathcal{B}_{\max}|^J$ channel assignment combinations. When the J or $|\mathcal{B}_{\max}|$ is large, the complexity can be very high. However, we can reduce the search complexity by exploring the special structure of the problem.

Property 3.1. *For each channel allocated positive power $i \in \mathcal{B}_p$, we have*

$$\mathcal{J}_i^* = \arg \max \left\{ Q_j \mid h_{ij} > \frac{\lambda}{Q_j \omega_i}, j \in \mathcal{J} \right\}. \quad (3.50)$$

This property comes from (3.48). This means that for a particular channel i , if the channel gain for the longest queue Q_j is good enough (*i.e.*, $h_{ij} > \frac{\lambda}{Q_j \omega_i}$),

we should assign channel i to queue with the longest queue length. If \mathcal{J}_i^* is a singleton, then we denote its unique element as j_i^* . Otherwise, we denote $\hat{\mathcal{J}}_i^* \triangleq \arg \max \{h_{ij} \mid j \in \mathcal{J}_i^*\}$. In this case, there are multiple channels with the same channel gain and the same queue length, and we can randomly pick one and denote it as j_i^* . Thus we can search λ and \mathcal{J}_i^* (and also j_i^*) by a simple greedy algorithm as follows. First, for all channels, we assume $\mathcal{J}_i^* = \arg \max_{j \in \mathcal{J}} Q_j$, and calculate the value of λ by the waterfilling algorithm (as the procedure “Waterfilling” in Alg. 7). For each unchosen channel, *i.e.*, the channel i with $\omega_i Q_{j_i^*} h_{ij_i^*} \leq \lambda$, we check whether $\omega_i Q_j h_{ij} > \lambda$ can be satisfied when another queue is chosen instead of j_i^* . If there is some set $\tilde{\mathcal{J}}_i$ of queues satisfying $\omega_i Q_j h_{ij} > \lambda$, we replace \mathcal{J}_i^* with the one with the longest queue length in this set, *i.e.*, $\arg \max_{j \in \tilde{\mathcal{J}}_i} Q_j$. We repeat the process iteratively until we find the λ and j_i^* that satisfies both (3.46) and (3.48). The pseudo code is given in Alg. 7. To simplify the expression of Alg. 7, with a little abuse of notations, we denote $h_i = h_{ij_i^*}$, $Q_i = Q_{j_i^*}$, and $\Lambda(m) \triangleq \frac{\sum_{i=1}^m \omega_i Q_i}{P_{\max} + \sum_{i=1}^m \frac{1}{h_i}}$.

The complexity of Alg. 7 is $\mathcal{O}(|\mathcal{B}_{\max}|^3 \log(|\mathcal{B}_{\max}|))$, since the while loop runs no more than $|\mathcal{B}_{\max}|$ times in the worst case, and the complexity of waterfilling part is $\mathcal{O}(|\mathcal{B}_{\max}|^2 \log(|\mathcal{B}_{\max}|))$ (the same as the waterfilling power allocation algorithm in Section 3.5.3.1). Compared to the exhaustive search, the complexity of solving the channel assignment is greatly reduced.

With the solution of channel assignment, we can update the power allocation solution of (3.43) as

$$P_{ij}^* = \begin{cases} \omega_i Q_j \left(\frac{1}{\lambda} - \frac{1}{\omega_i Q_j h_i} \right)^+, & \text{if } j = j_i^*, i \in \mathcal{B}, \\ 0 & \text{otherwise,} \end{cases}$$

where the value of λ , h_i and Q_i are calculated by Alg. 7.

Algorithm 7 Channel Assignment

- 1: $\mathcal{J}_i^* \leftarrow \arg \max_{j \in \mathcal{J}} Q_j$

 - 2: **procedure** WATERFILLING($h_i(t), Q_i(t)$)
 - 3: Rearrange the channel indices $i \in \mathcal{B}_{\max}$ in the decreasing order of $\omega_i(t)Q_i(t)h_i(t)$
 - 4: $m \leftarrow |\mathcal{B}(t)|, \lambda \Leftarrow \Lambda(m)$
 - 5: **while** $\lambda \geq h_m(t)\omega_m(t)$ **do**
 - 6: $m \leftarrow m - 1$
 - 7: $\lambda \Leftarrow \Lambda(m)$
 - 8: **end while**
 - 9: **end procedure**

 - 10: **while** $\omega_i(t)h_{ij}(t)Q_j(t) > \lambda, \forall j, \forall i > m$, **do**
 - 11: $\mathcal{J}_i^* \leftarrow \arg \max \{Q_j | \omega_i(t)h_{ij}(t)Q_j(t) > \lambda\}$
 - 12: **invoke** procedure Waterfilling($h_i(t), Q_i(t)$)
 - 13: **end while**
-

After solving the second stage problem, we move to the first stage. Following the channel assignment in (3.49), we find the first stage problem for the heterogeneous users is the same with the one of homogeneous users problem in (3.30). We can simply run the same Alg. 6 to determine sensing technology $C^{s*}(t)$, sensing channel set $\mathcal{B}^{s*}(t)$ and leasing channel set $\mathcal{B}^{l*}(t)$.

3.6.2 Performance of the M-PMC Policy

Next we show the performance bounds of the M-PMC Policy. The proof method is similar to that of Theorem 3.1, and the details are omitted due to space limit.

Theorem 3.2. *For any positive value V , the M-PMC Policy has the following properties:*

- (a) *The queue stability (3.11) and collision constraints (3.12) are satisfied. The queue length is upper bounded by*

$$Q_j(t) \leq Q^{\max} \triangleq Vq^{\max} + A^{\max}, \quad \forall t, \quad (3.51)$$

and the virtual queues are bounded by

$$Z_i(t) \leq Z^{\max} \triangleq \kappa(Vq^{\max} + A^{\max}) + 1, \quad \forall i, t. \quad (3.52)$$

where $\kappa \triangleq r_{\max} p_0 \frac{p_0(1-P_{fa}(C^s(t)))+(1-p_0)P_{md}(C^s(t))}{(1-p_0)}$, and C^{s0} denotes the highest sensing cost.

- (b) *The average profit $\overline{R_{M-PMC}}$ obtained by the M-PMC policy satisfies*

$$\inf \overline{R_{M-PMC}} \geq \overline{R}^* - O(1/V), \quad (3.53)$$

where \overline{R}^ is the optimal value of the multi-queue PM problem.*

3.7 Simulation

In this section we provide simulation results for PMC and M-PMC policies.

We conduct simulations with the following parameters. The number of incoming users in each slot satisfies a Poisson distribution with a rate $D(q(t), M(t)) = \frac{1}{M(t)}(q(t) - 5)^2$. The market state $M(t)$ satisfies Bernoulli distribution, $M(t) = 1$ with probability 0.5, and $M(t) = 2$ with probability 0.5. The file length of each user satisfies the i.i.d. (discrete) uniform distribution between 1 and 10. There are 32 channels in total. 20 of them belong to the sensing band \mathcal{B}_{\max}^s , and the rest 12 channels belong to the leasing band \mathcal{B}_{\max}^l . The primary collision probability tolerant levels are set as $\eta_i = 0.001$ for sensing channel $i = 1, 2, \dots, 10$, and $\eta_i =$

0.005 for sensing channel $i = 11, 12, \dots, 20$. The channel gain h_i of each channel satisfies i.i.d. (continuous) Rayleigh distribution with parameter $\sigma = 4.5$. The total power constraint of the base station is $P^{\max} = 8$. There are 3 different sensing technologies with costs $\mathcal{C}^s = \{0 \text{ (not sensing at all), } 0.1, 0.5\}$. The corresponding false alarm probabilities are $P_{fa} = \{0.5, 0.1, 0.008\}$, and the missed detection probabilities are $P_{md} = \{0.5, 0.08, 0.005\}$. We assume the idle time probability of sensing band p_0 is 0.6, and the control parameter $V \in \{5, 10, 50, 100, 200\}$.

Figure 3.5 shows a collision situation of all sensing channels with the control parameter $V = 100$. We find that primary users' collision tolerant bound (3.12) is satisfied as time increases. We also find that we obtain similar curves for the collision probabilities with other values of control parameter V .

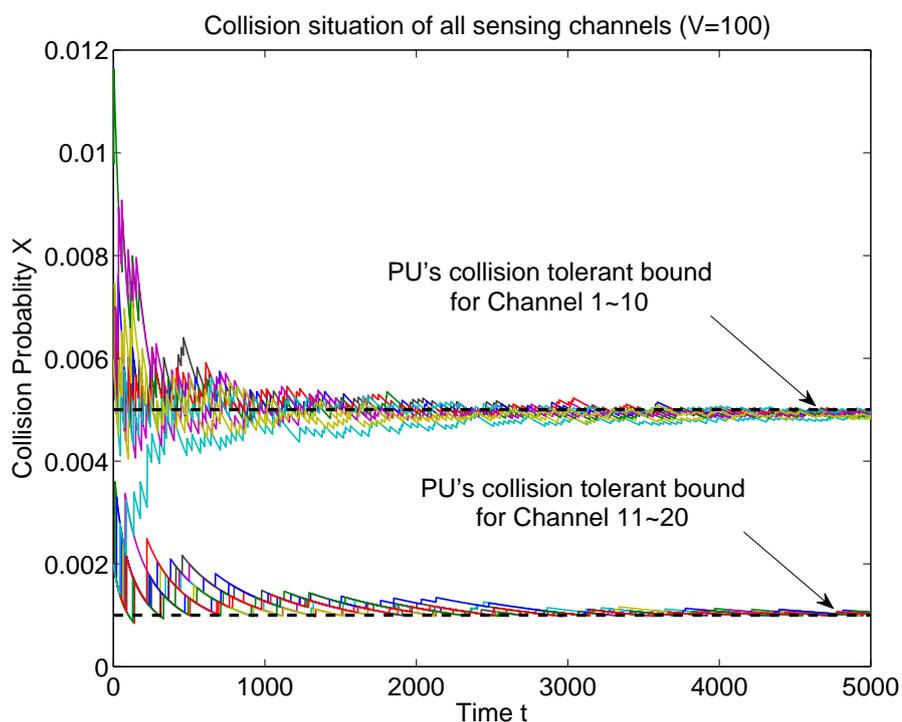


Figure 3.5: A collision situation of all sensing channels with $V = 100$

Figure. 3.6 (a) shows that the average queue length grows linearly in V , and is always less than the worst case bound $Vq^{max} + A^{max}$. Figure. 3.6 (b) shows that the average profit achieved by PMC policy converges quickly as V grows, and is close to the maximum profit when $V \geq 100$.

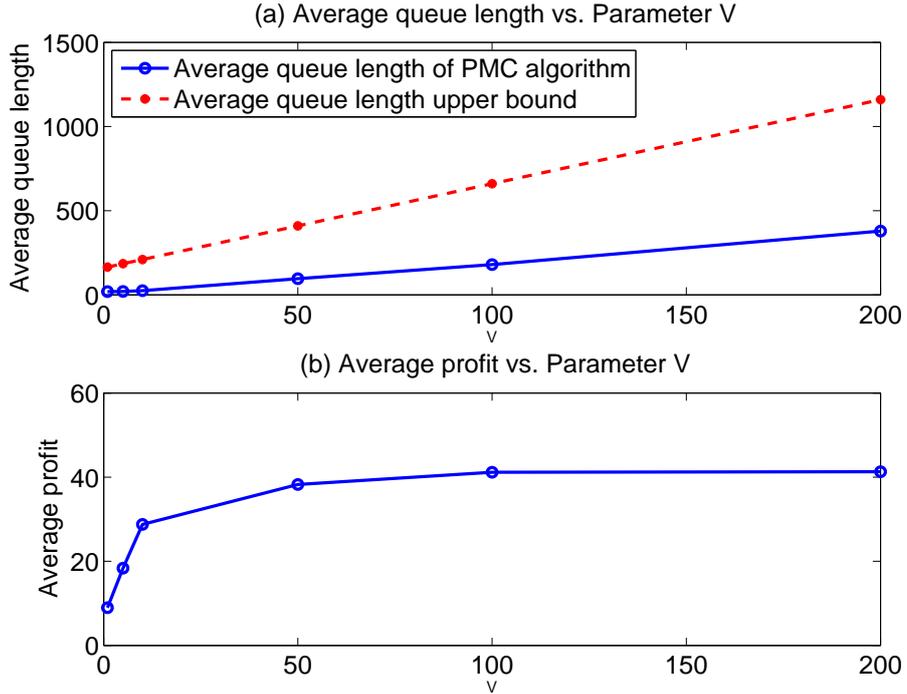


Figure 3.6: (a) Average queue length vs. Parameter V , (b) Average profit vs. Parameter V

We further vary the idle time probability of sensing band p_0 from 0 to 1. In Fig. 3.7, we show the average profit with different sensing available probabilities $p_0 \in [0, 1]$ and different fixed sensing technologies. The black curve is with zero sensing cost, where $P_{fa} = P_{md} = 0.5$, which means the operator makes no sensing and takes random guesses of primary users' activities in sensing channels. The blue curve is with the low sensing cost 0.1, where $P_{fa} = 0.1$ and $P_{md} = 0.08$. The purple curve is with the high sensing cost 0.5, where $P_{fa} = 0.008$ and

$P_{md} = 0.005$. The red curve is the PMC policy, which adaptively choose sensing cost from the above three (sensing cost $C_s = 0, 0.1, 0.5$). When the sensing available probability is small (e.g., $p_0 \in [0, 0.2]$), all strategies tend to only choose the leasing channels, thus all curves obtain similar profits. When the sensing available probability p_0 further increases, the advantage of exploring sensing channels becomes more significant. The performances for strategies using the zero and low sensing cost are not good. The reason is that their detection accuracy is not good enough. To achieve the primary users' collision bounds, these strategies choose sensing channels less often, and replace with more expensive leasing channels. When the sensing available probability is high and close to 1, sensing seems unnecessary. Therefore, the performance increases as sensing cost decreases, where the zero cost is the best and the high cost is the worst. The PMC policy (the red curve) adaptively chooses the sensing cost, *i.e.*, when p_0 is medium, it utilizes the high sensing cost strategy in most of time slots; as p_0 keeps increasing, it gradually changes to utilize the low cost and zero cost strategies more frequently; and when p_0 goes to 1, it utilizes the zero sensing cost strategy in most of time slots. It has the best performance, since it can take advantage of different sensing technologies for different sensing available probabilities.

For the M-PMC policy, we conduct simulations for a simple two-queue system. For queue-1, the channel gain h_{i1} of each channel satisfies i.i.d. (continuous) Rayleigh distributions with parameter $\sigma = 4.5$. For queue-2, the channel gain h_{i2} of each channel satisfies i.i.d. (continuous) Rayleigh distributions with parameter $\sigma = 5.5$. This is because queue-2 users are nearer to the operator's base station than queue-1 users.

Figure 3.8 (a) shows that the average transmission rate obtained by queue-2 users is higher than that of the queue-1 users. This is because queue-2 users usu-

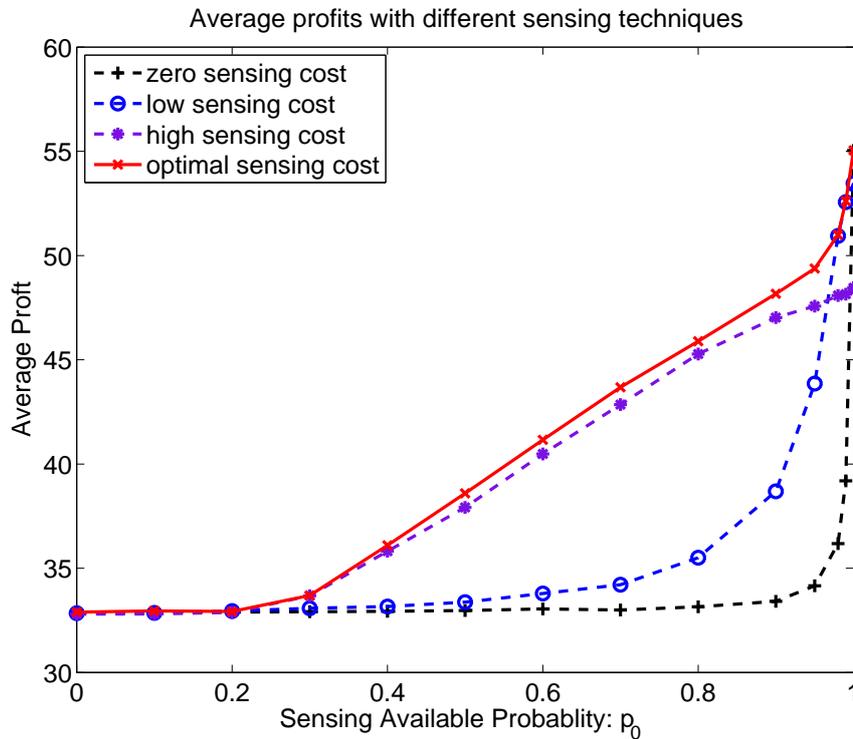


Figure 3.7: Average Profit with different sensing technologies

ally have better channel conditions, and M-PMC policy prefers to allocate more powers to better channels to improve the transmission rate. Figure 3.8 (b) shows that the revenues obtained by the operator from users of two queues are almost the same when all queues are stable. It is an interesting observation. We can understand it in this way: when two queues make different revenue, to maximize the profit (also the revenue), the operator will allocate more transmission rates to the queue with a higher revenue. Thus the length of the queue with a higher transmission rate will be shorten, and the negative queuing effect in revenue maximization problem will be soon diluted. It further leads to a decreasing price and a decreasing revenue. In contrast, the length of the queue with a lower transmission rate increases, which results in an increasing price and an increasing revenue. There-

fore, when all queues are stable finally, the average revenue generated by each queue is the same.

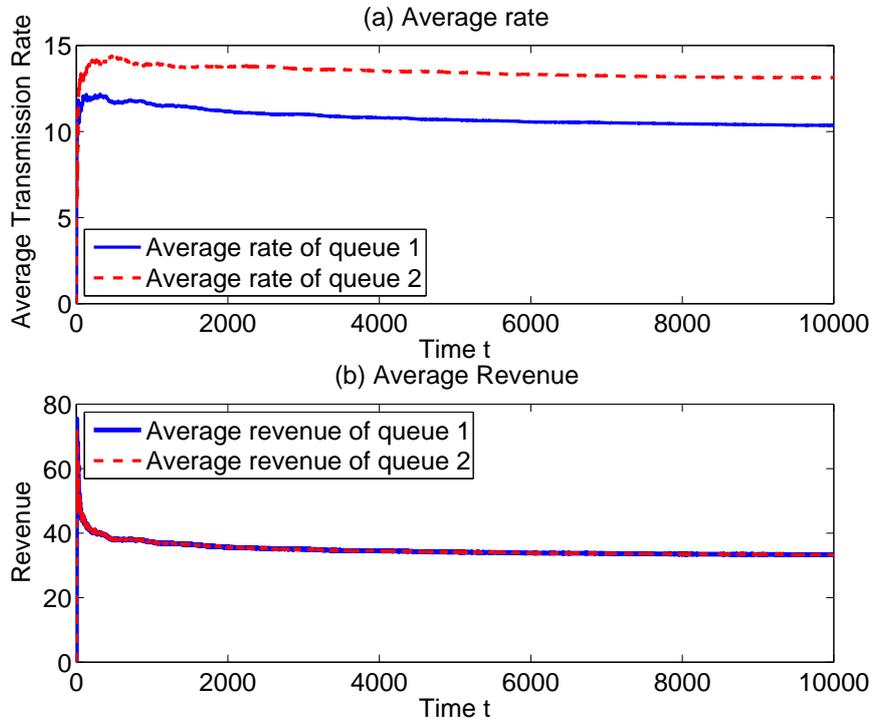


Figure 3.8: (a) Average transmission rates of a two-queue M-PMC policy, (b) Average revenues of a two-queue M-PMC policy

3.8 Summary

In this chapter, we study the profit maximization problem of a cognitive mobile virtual network operator in a downlink OFDM transmission system with an uncertain and dynamic network environment. By exploring the special structural information of the problem, we propose low-complexity PMC and M-PMC policies which perform both revenue maximization with pricing and market control, and cost minimization with proper resource investment and allocation. We show

that these policies can achieve arbitrarily close to the optimal profit, and have flexible trade-offs between profit optimality and queuing delay. We also find several interesting features in these close-to-optimal policies. In revenue maximization, the dynamic pricing strategy performs the functionality of congestion control to users' demands, *i.e.*, the longer the queue length of demands, the higher price the operator should charge. In cost minimization, the operator shows risk aversion in spectrum investment, which prefers stable leasing spectrum to unstable sensing spectrum with the same channel condition and the same availability-price-ratio.

3.9 Appendix of Chapter 3

3.9.1 (Waterfilling) Power Allocation Algorithm

We define the function $\Lambda(m)$ as (3.27):

$$\Lambda(m) \triangleq \frac{\sum_{i=1}^m \omega_i}{P_{\max} + \sum_{i=1}^m \frac{1}{h_i}}.$$

The pseudo code of the power allocation algorithm is shown as follows.

Algorithm 8 Power Allocation

- 1: Rearrange the channel indices $i \in \mathcal{B}_{\max}$ as a decreasing order of $\omega_i(t)h_i(t)$
 - 2: $m \leftarrow |\mathcal{B}(t)|$, $\lambda \leftarrow \Lambda(m)$
 - 3: **while** $\lambda \geq h_m(t)\omega_m(t)$ **do**
 - 4: $m \leftarrow m - 1$
 - 5: $\lambda \leftarrow \Lambda(m)$
 - 6: **end while**
-

3.9.2 Threshold Searching Algorithm

The pseudo code of threshold searching algorithm is shown in Algorithm 9.

Algorithm 9 Optimal Channel Selection (for a given $C^s(t)$)

```

1: procedure COMPUTING  $\mathcal{B}(C^s(t))$ 
2:   invoke procedure SearchingThreshold( $C^s$ ) to calculate  $\Upsilon^l$  and  $\Upsilon^s$ 
3:    $U(C^s(t)) \leftarrow 0$ 
4:   for  $i = 0, 1, \dots, \Upsilon^l$  do
5:     for  $j = 0, 1, \dots, \Upsilon^s$  do
6:       Calculate  $\lambda(t)$  as (3.27) with  $\mathcal{B}_p(t) = \mathcal{B}_i^l \cup \mathcal{B}_j^s$ 
7:       if  $g_i(t) > \lambda(t)$  and  $g_j(t) > \lambda(t)$  then
8:         Calculate  $U(i, j)$ 
9:         if  $U(i, j) < U(C^s(t))$  then
10:            $U(C^s(t)) \leftarrow U(i, j)$ 
11:            $\mathcal{B}(C^s(t)) \leftarrow \mathcal{B}_i^l \cup \mathcal{B}_j^s$ 
12:         end if
13:       end if
14:     end for
15:   end for
16: end procedure

```

In Algorithm 9, $U(i, j)$ denotes the optimal value of (3.34) with the channel selection set $\mathcal{B} = \mathcal{B}_i^l \cup \mathcal{B}_j^s$. To decrease the number of searching loops, we can first run Algorithm 10 to determine the maximum possible thresholds Υ^l for leasing channels or Υ^s for sensing channels. The illustration of the maximum possible threshold is given in Fig. 3.9. If we do not run Algorithm 10, we can just set $\Upsilon^l = |\mathcal{B}_{\max}^l|$ and $\Upsilon^s = |\mathcal{B}_{\max}^s|$. Whether we run Algorithm 10 or not, the complexity of Algorithm 9 can not be worse than $\mathcal{O}(|\mathcal{B}_{\max}^s| \times |\mathcal{B}_{\max}^l|)$.

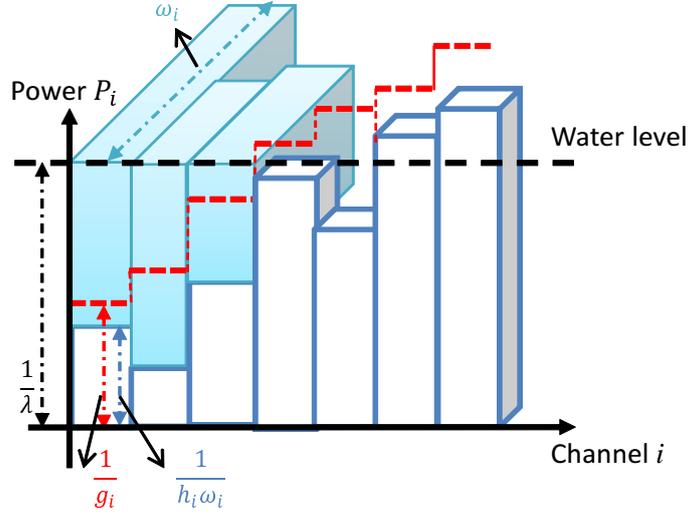


Figure 3.9: An illustration for Alg. 10. The maximum possible threshold is 3.

3.9.3 Proof for Theorem 3.1 (a)

We first prove (3.35) by induction.

It is easy to see that at slot $t = 0$, no packets are in the network and $Q(0) = 0$, thus the queue length bound (3.35) obviously holds. Now suppose (3.35) holds for time t . We consider the queue length bound in slot $t + 1$ in the following cases:

- *Case 1:* $Q(t) \leq Vq_{\max}$, then clearly $Q(t + 1) \leq Vq_{\max} + A_{\max}$.
- *Case 2:* $Q(t) > Vq_{\max}$, *i.e.*, the objective value of the revenue maximization (3.21) is negative. Therefor, according to the optimization solution, the operator will set $O(t) = 0$, and do not accept any new request, $A(t) = 0$. Therefore, $Q(t + 1) \leq Q(t) \leq Vq_{\max} + A_{\max}$.

Likewise, we can also prove the virtual queue bound (3.36) by induction. Suppose that the inequality (3.36) holds for time t , and consider the following two cases.

- *Case 1:* $Z_i(t) \leq Z_{\max} - 1$, clearly the virtual queue length bound (3.36)

Algorithm 10 Search Threshold Υ^l (or Υ^s) for a given $C^s(t)$

```

1: procedure SEARCHINGTHRESHOLD( $C^s$ )
2:   Rearrange the channel indecent  $i \in \mathcal{B}_{\max}^l$  (or  $i \in \mathcal{B}_{\max}^s$ ) as the decreasing
   order of  $g_i(t)$ .
3:    $m \leftarrow |\mathcal{B}_{\max}^l|$  (or  $m \leftarrow |\mathcal{B}_{\max}^s|$ ),  $\lambda \Leftarrow \Lambda(m)$ 
4:   while  $\lambda \geq g_m(t)$  do
5:     if  $m > 1$  then
6:        $m \leftarrow m - 1$ 
7:        $\lambda \Leftarrow \Lambda(m)$ 
8:     else
9:       break
10:    end if
11:  end while
12:   $\Upsilon^l \leftarrow m$  (or  $\Upsilon^s \leftarrow m$ )
13: end procedure

```

also holds at slot $t + 1$.

- *Case 2:* $Z_i(t) > Z_{\max} - 1$, *i.e.*,

$$Z_i(t) \geq \max_{C^s} \frac{r_{\max} Q_{\max} \alpha_i(t)}{(1 - p_0) P_{md}(C^s(t))} \quad (3.54)$$

Since $r_{\max} \geq \sum_{i \in \mathcal{B}_{\max}} r_i(t) = \sum_{i \in \mathcal{B}_{\max}} \log \left(\frac{\omega_i(t) h_i(t)}{\lambda(t)} \right)$, then inequality (3.54) implies

$$C_i(t) = C^s(t) + Z_i(t) \frac{1 - p_0}{V} > \alpha_i(t) \log \left(\frac{\omega_i(t) h_i(t)}{\lambda(t)} \right). \quad (3.55)$$

By (3.33) in PMC policy, channel i will not be chosen for sensing and transmission, thus there will be no collision in this channel, *i.e.*, $X_i(t + 1) = 0$. Then by (3.13) we have $Z_i(t + 1) \leq Z_i(t)$ and the virtual queue length bound (3.36) also holds at $t + 1$.

3.9.4 Proof for Theorem 3.1 (b)

We first construct a **stationary** randomized policy that can achieve the optimal solution of the PM problem. Let us consider a special class of stationary randomized policies, called *ϕ -only policy*, which makes the decision $\gamma(t)$ in slot t only depending on the observation of system parameter $\phi(t)$. The stationary distribution for the observable parameter $\phi(t)$ is denoted as $\{\Pi_\phi, \phi \in \Phi\}$. (Remember we define $\phi(t) \triangleq (M(t), \mathbf{h}(t), C^l(t))$, $\gamma(t) \triangleq (O(t), q(t), C^s(t), \mathcal{B}^s(t), \mathcal{B}^l(t), \mathbf{P}(t))$. Note that the value of $\phi(t)$ can be chosen only from a finite set Φ .) In the ϕ -only policy, when the operator observes $\phi(t) = \phi$, it chooses $\gamma(t)$ from the countable collection of $\Gamma_\phi(t) = \{\gamma_\phi^1, \gamma_\phi^2, \dots\}$ with probabilities $\{\rho_\phi^1, \rho_\phi^2, \dots\}$, where $\sum_{u=1}^{\infty} \rho_\phi^u = 1$. Note that the decision is independent of time t , and thus is stationary. We have the following fact:

There exists a stationary ϕ -only policy that achieves the optimal profit of the PM problem while satisfying stability condition (3.11) and collision upper-bound requirement (3.12), which is the solution of the following optimization problem:

$$\begin{aligned} \bar{R}^* = \text{Maximize} \quad & \sum_{\phi \in \Phi} \Pi_\phi \sum_{u=1}^{\infty} R(\gamma_\phi^u; \phi) \rho_\phi^u \\ \text{Subject to} \quad & \sum_{\phi \in \Phi} \Pi_\phi \sum_{u=1}^{\infty} r(\gamma_\phi^u; \phi) \rho_\phi^u \leq \sum_{\phi \in \Phi} \Pi_\phi \sum_{u=1}^{\infty} D(\gamma_\phi^u; \phi) \rho_\phi^u \\ & \sum_{\phi \in \Phi} \Pi_\phi \sum_{u=1}^{\infty} X_i(\gamma_\phi^u; \phi) \rho_\phi^u \leq \eta_i, \quad i \in \mathcal{B}_{max}^s \end{aligned}$$

The above fact is a special case of Theorem 4.5 in [21]. The proof is omitted for brevity.

Recall that the PMC policy is derived by minimizing the right hand side of the

following inequality (it is actually (3.18))

$$\begin{aligned} \Delta(\Theta(t)) - V\mathbb{E}[R_{PMC}(t)|\Theta(t)] &\leq D - V\mathbb{E}[R(t)|\Theta(t)] \\ &+ Q(t)\mathbb{E}[O(t)A(t) - r(t)|\Theta(t)] \\ &+ \sum_{i \in \mathcal{B}_{\max}^s} Z_i(t)\mathbb{E}[X_i(t) - \eta_i|\Theta(t)]. \end{aligned} \quad (3.56)$$

In other words, given the current queue backlogs for each slot t , the PMC policy minimizes the right hand side of (3.56) over all alternative feasible policies that could be implemented, including the optimal stationary ϕ -only policy. Therefore, by plugging the optimal stationary ϕ -only policy in the right hand side of (3.56), we have

$$\Delta(\Theta(t)) - V\mathbb{E}[R_{PMC}(t)|\Theta(t)] \leq D - V\bar{R}^*. \quad (3.57)$$

Now we use the following lemma to obtain the performance bound in Theorem 3.1(b).

Lemma 3.1. (*Lyapunov Optimization*) *Suppose there are finite constants $V > 0$, $D > 0$, such that for all time slots $t \in \{0, 1, 2, \dots\}$ and all possible values of $\Theta(t)$, we have*

$$\Delta(\Theta(t)) - V\mathbb{E}[R(t)|\Theta(t)] \leq D - V\bar{R}^*. \quad (3.58)$$

Then we have the following result

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[R(\tau)] \geq \bar{R}^* - \frac{D}{V}. \quad (3.59)$$

The above lemma is a special case of Theorem 4.2 in [21]. The proof is omitted for brevity.

Note that the inequality (3.57) is exact the condition (3.58) in Lemma 3.1, thus the performance bound in Theorem 3.1(b) immediately follows.

3.9.5 Impact of Queueing on Revenue Maximization

What is the impact of the queueing effect on the pricing in the revenue maximization problem (3.21)? Let's consider the following instantaneous revenue maximization problem without the queueing shift.

$$\text{Maximize } qD(q, M). \quad (3.60)$$

For simplicity, we ignore the time index in the discussion.

Note that both problems in (3.21) and (3.60) may have multiple optimal solutions. For the purpose of obtaining intuitions, we will restrict our discussion to the case where there is a unique optimal price for both (3.21) and (3.60). To guarantee this, we assume that revenue $R(D) \triangleq \hat{q}(D)D$ is a strictly concave function of the demand⁵, where $\hat{q}(D)$ is defined as the inverse demand function, *i.e.*, $\hat{q}(D) \triangleq \max\{\hat{q} : D(\hat{q}, m) = D\}$ ⁶ for a given m . For simplicity, we denote the optimal price in revenue maximization (3.21) as q^* , and the optimal price in revenue maximization problem (3.60) with the queueing shift as q^{**} . We will show that $q^{**} \geq q^*$, *i.e.*, the queueing effect leads to a higher price.

The objective of revenue maximization problem in (3.60) can be represented as $R(D) = \hat{q}(D)D$, and its optimal demand is denoted as D^* . By the first order optimality condition, D^* satisfies that $R'(D^*) = 0$, where $R'(\cdot)$ denotes the first order derivative of $R(\cdot)$. When the queueing effect is taken into consideration, the objective of (3.21) can be represented as $R(D) - \frac{DQ}{V}$, and we denote its optimal demand as D^{**} . Again by the first optimality condition, D^{**} satisfies that

⁵This assumption is common in the revenue management literature (*e.g.*, [43]) to guarantee unique optimal pricing.

⁶Since the demand function $D(q, m)$ is non-increasing in q , there can be multiple prices resulting the same demand.

$R'(D^{**}) = \frac{Q}{V} \geq 0$. Since the revenue function $R(D)$ is concave in D , $R'(D)$ is a decreasing function. Since $R'(D^{**}) \geq R'(D^*)$, we obtain $D^{**} \leq D^*$. Furthermore, since the demand $D(q, m)$ is non-increasing in price q for a given m , we have $q^{**} \geq q^*$. In other words, when incorporating the queuing effect, the optimal dynamic price q^{**} in the PMC policy is higher than the optimal price q^* in instantaneous revenue maximization problem without the shift. Moreover, the larger the queue length Q , the higher the dynamic price q in the PMC policy. When we perform such pricing in the system, a high price will decrease the demand, which will slow the increase of the queue length. Thus the dynamic pricing in the PMC policy also performs the functionality of congestion control to some extent.

For a more general case where the concavity assumption may not be satisfied, the queuing effect depends on the shape of the revenue function at the point D^* , *i.e.*, $q^{**} \geq q^*$ if and only if $R'(D^*) \leq \frac{Q}{V}$.

Chapter 4

Distributed Resource Allocation for Node-Capacitated Networks with Network Coding

Node-capacitated networks are those networks with the capacity constraints of every node. In this chapter, we consider a node-capacitated multicast network with a time-varying topology. By utilizing network coding, we design a dynamic and distributed pricing and resource allocation algorithm that can achieve arbitrarily close to maximum network utility while maintaining the network stability. In addition, we show that this algorithm can provide incentives for nodes to stay in the network and relay traffic for others even when they do not have interested contents.

4.1 Node-Capacitated Networks

For a long time, the bottleneck of communication networks is the link bandwidth between any two network nodes. Therefore, most previous studies in communi-

ation network adopt the link-capacitated network model. However, due to the development of the broadband communication technologies in recent years, node capacity constraint becomes an essential issue that can not be ignored in many important network applications:

- **P2P Networks** are constructed by computers connected through Internet. In such networks, bottlenecks typically are at the side of the access networks rather than in the middle of the Internet (see [91–94]). Therefore, when modeling P2P networks, capacity constraints are usually put on nodes (the connection of the computer to the Internet) rather than links between nodes.
- **Optical Networks** are networks supported by fiber links between each nodes. Since each fiber link has a huge bandwidth, the link capacity can be regarded as infinity. In contrast, the nodes in optical networks usually have capacity constraints due to many technique limitations in Optical-Electronic-Optical (OEO) conversions or all-optical conversions, wavelength conversions, and wavelength-continuity constraints.

What is more, in several wireless applications such as wireless sensor networks, since the nodes in the these networks usually have limited energy supplies, total flow in and out of these nodes are also constrained. In this way, these wireless networks can be regarded as networks with both link capacity constraints and node capacity constraints.

Besides these important practical applications, the node-capacitated networks also provide a more general network model. It is well known that a link-capacitated network can be easily converted to an equivalent node-capacitated network [95]. However, the conversion on the other way round is not feasible in general.

To see these, let us consider a network presented by a directed graph $G = (V, E)$, where V is the set of vertices, *i.e.*, nodes in the network, and E is the

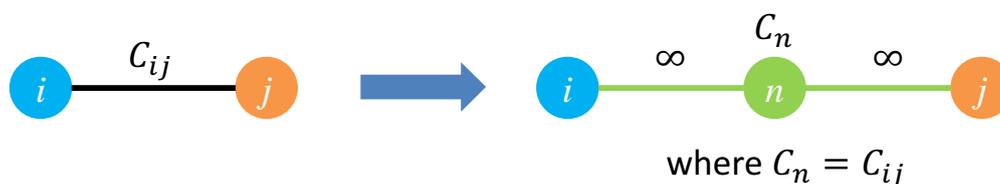


Figure 4.1: Standard transformation from a link-capacitated network to node-capacitated network

set of edges, *i.e.*, links in the network. Then an edge-capacitated network can be represented by $G = (V, E)$ with an edge capacity function $C : E \rightarrow \mathbb{R}^+$. Similarly, a node-capacitated network can be represented by $G = (V, E)$ with a node capacity function $C : V \rightarrow \mathbb{R}^+$.

We have the following standard transformation [95] converting a link-capacitated network to a corresponding node-capacitated network, which is illustrated in Fig.4.1. Given an edge-capacitated network G , we first subdivide each edge ij with a new node n , then assign the amount of edge capacity C_{ij} to the node capacity of n . Both edge in and edge nj have infinite capacities. Therefore, we can get the corresponding node-capacitated network \tilde{G} .

But finding a general method to convert node-capacitated networks to their link-capacitated counterparts is still open. People once believed that the following transformation [95, 96] is the answer, which is illustrated in Fig.4.2. Given a node-capacitated network G , just split each node i with node capacity C_i into two incapacitated nodes i^+ and i^- , then introduce a directed link from i^+ to i^- with capacity $C_{i^+i^-} = C_i$. We put all the incoming links of i to i^+ and make all the outgoing edges of i leave from i^- (all these edges have capacities ∞). Thus we obtain an edge-capacitated network \tilde{G} . This transformation does work for some particular problems, for example, deriving the node-capacitated Max-

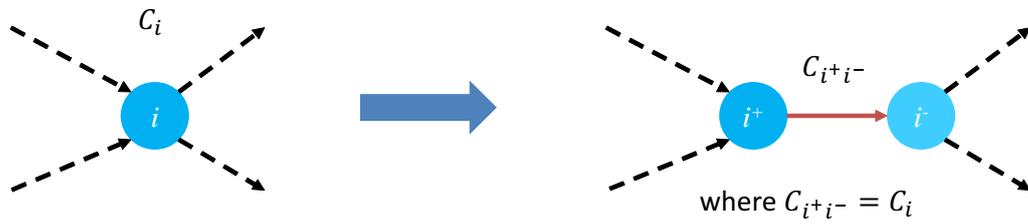


Figure 4.2: Transformation From Node-capacitated Networks to Edge-capacitated Networks

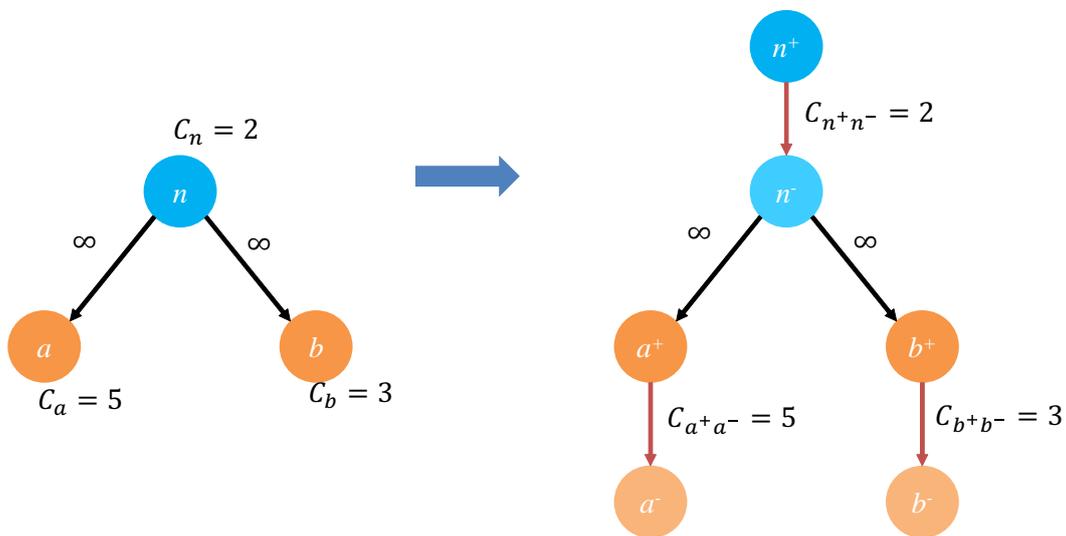


Figure 4.3: An example of the transformation fails in the broadcast scenario

Flow-Min-Cut theorem [95] (this theorem was first proved for edge-capacitated networks). However, it does not work in general [96], since the transformation may create some properties that the initial problem does not have. For example, in the Fig.4.3, the node-capacitated network on the left can only support a broadcast rate of one, but the corresponding edge-capacitated network on the right can support a broadcast rate of two. The problem arises because the transformation ‘creates’ additional capacity when there are multiple receivers sharing the same flow paths. Therefore, the node-capacitated network model is a generalization of

the link-capacitated network model. Existing results with the link-capacitated network model can not always be extended to their node-capacitated counterparts in a straightforward way.

In this chapter, we consider a node-capacitated multicast network with a general time-varying topology and we apply the network coding technology [97–99] to this network. We propose a dynamic and distributed algorithm that approaches the maximum network utility arbitrarily closely with network stability. Several existing literatures can be viewed as special cases of our work, as they all study static scenarios, either the broadcast scenario (*e.g.*, [91–93, 100]) or multicast scenario under a special network topology (*e.g.*, [94]).

Note that we consider the time-varying network topology, which means that nodes have the freedom to join and leave the network. We show that our proposed algorithm can provide nodes good incentives to stay in the network and relay traffic for other nodes, even when they are not interested in any content themselves. This is different from the “tit-for-tat” incentive strategy, *i.e.*, downloading more only when uploading more, which has been intensively studied in P2P system (*e.g.*, [101, 102]).

We propose the dynamic and distributed pricing and resource allocation algorithm based on the Lyapunov stochastic optimization. We also apply the network coding technique [97–99], which can further improve the network throughput and achieve the optimality beyond the traditional routing technique in many network scenarios, including the mulitcast scenario discussed in this chapter.

The key contributions of this work include:

- *General Network Model:* We consider a multi-source multicast scenario with a general network topology in a node-capacitated network. Our analysis is based on a packet level network model, which incorporates various

transmission details and network dynamics and thus better approximates the real networks than the fluid model (*e.g.*, [91–94, 100]).

- *Distributed Algorithm Design:* We design a dynamic and distributed algorithm for resource allocation in node-capacitated networks. The proposed algorithm has a low complexity, and can achieve arbitrarily close to maximum network utility while maintaining network stability. We further characterize the trade-off between utility optimality and queuing delay of the algorithm.
- *Incentive Mechanism:* We show that the proposed algorithm can motivate nodes to join and stay in the network, even when they are not interested in the contents themselves. The reason behind this is that the algorithm can guarantee non-negative profits for them for relaying other nodes' contents.

The rest of chapter is organized as follows. We present the network model in Section 4.2. We formulate the utility maximization problem in Section 4.3 and design a low-complexity distributed algorithm in Section 4.4. We prove the performance bound of the algorithm and its incentive compatibility in Section 4.5, and summarize this chapter in Section 4.6.

4.2 Network Model

4.2.1 Time-varying network topology and node upload capacities

We consider a time-slotted node-capacitated network including a set $\mathcal{N} \triangleq \{1, 2, \dots, N\}$ of nodes. During time slot $t \in \{0, 1, 2, \dots\}$, the network topology can be characterized by a matrix $\mathbf{S}(t) = [S_{ab}(t)]_{a,b \in \mathcal{N}}$, where $S_{ab}(t) = 1$ if there is a link from

node a to node b at time t , and $S_{ab}(t) = 0$ otherwise. The link capacities during time slot t is $r(\mathbf{S}(t)) = [r_{ab}(S_{ab}(t))]_{a,b \in \mathcal{N}}$, which depend on the network topology, *i.e.*, $r_{ab}(t)$ equals to the maximum capacity of link ab at time t if $S_{ab}(t) = 1$ and is 0 otherwise. We further assume that $\mathbf{S}(t)$'s are independently and identically distributed over time, with the set of all feasible values as \mathcal{S} .

In a node-capacitated network, each node consumes resources by downloading contents from other nodes and provides resources by uploading contents to other nodes. We consider the most general case that each node has a limited and time-varying upload capacity. A practical example can be the P2P application on a computer sharing the Internet connection with many other applications simultaneously. We denote all nodes' upload capacities during time slot t as $\mathbf{J}(t) = [J_n(t)]_{n \in \mathcal{N}}$. For simplicity, we assume that $\mathbf{J}(t)$'s are independently and identically distributed over time¹, with the set of all feasible values as \mathcal{J} . As several node-capacitate applications (*e.g.*, the P2P application) today provide customers the freedom to choose their maximum amount of contribution, we use \bar{J}_n to denote node n 's choice of the maximum upload capacity, which is a constant parameter determined by the node independent of the instantaneous physical capacity $J_n(t)$. We further assume that the nodes' download capacities are infinite.

For modeling the node-capacitated networks, we adopt the assumption for each node n , $r_{nb}(t) \gg J_n(t)$ for any node b with $S_{nb}(t) = 1$, *i.e.*, when there is a link from node n to node b , the link capacity is always larger than node n 's upload capacity. Finally, we assume that the network topology $\mathbf{S}(t)$ and nodes upload capacities $\mathbf{J}(t)$ only change at the beginning of each time slot and remain fixed

¹The i.i.d. assumption of $\mathbf{S}(t)$ and $\mathbf{J}(t)$ simplifies analysis. However, all results in this chapter can be extended to the case of general ergodic channel processes, using the T-slot Lyapunov drift in [21] Section 4.9.

during the time slot. For simplicity, we will use the notation $M(t) \triangleq (\mathbf{S}(t), \mathbf{J}(t))$ to denote the network topology and nodes' upload capacities at time t .

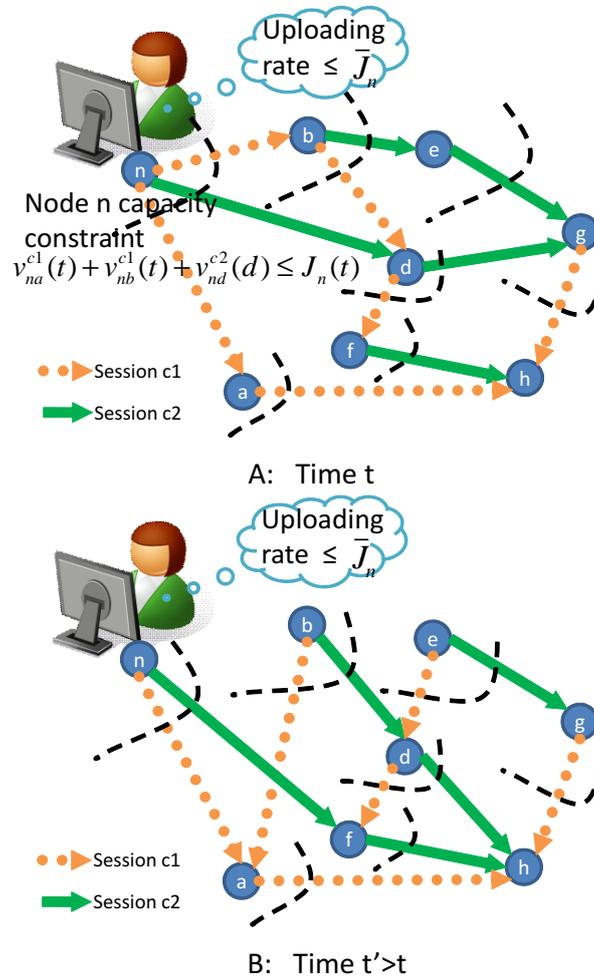


Figure 4.4: A general model for a dynamic node-capacitated networks

4.2.2 Multicast with intra-session network coding

There is a set \mathcal{C} of multicast sessions sharing the network. Each multicast session $c \in \mathcal{C}$ is associated with a source user $s_c \in \mathcal{N}$ and a set $\mathcal{T}_c \in \mathcal{N} \setminus s_c$ of sink users. It is well-known (e.g., [97]) that a multicast rate μ from a source s_c to a set

of receivers \mathcal{T}_c is achievable if and only if μ is feasible for s_c to any receiver $\beta \in \mathcal{T}_c$, and the maximum multicast rate in general can only be achieved by network coding instead of routing only. Ho et al. in [103] proposed an intra-session linear network coding scheme for distributed algorithm design for link-capacitated networks, and showed for enough large coding field \mathbb{F}_q and sufficient large time, the decoding probability is 1.

To maximize the multicast rate, we perform an intra-session linear network coding scheme as proposed in [103]: each node n maintains an “information” queue $U_n^{c\beta}$ for each session $c \in \mathcal{C}$ and each sink node $\beta \in \mathcal{T}_c$. At node n , a single *physical* packet of session c corresponds to a *virtual* packet in the information queue $U_n^{c\beta}$ of each sink $\beta \in \mathcal{T}_c$, *i.e.*, the mapping is one-to-many. By constructing the information queues, we can decompose each *physical* multicast session into multiple *virtual* unicast sessions. This significantly simplifies the distributed algorithm design later. With a little abuse of the notation, we use $U_n^{c\beta}(t)$ to denote the length (*i.e.*, the number of packets) of queue $U_n^{c\beta}$ at time t .

Next we characterize the dynamics of the information queue lengths. At a node n during time slot t , we denote $x_{s_c}^c(t)$ as the number of physical packets of session $c \in \mathcal{C}$ injected into the network by source node s_c , $v_{nb}^c(t)$ as the number of *physical* packets of session c transmitted on the link from node n to node b , and $\mu_{nb}^{c\beta}(t)$ for $\beta \in \mathcal{T}_c$ as the instantaneous *information* rate of session c transmitted on the link from node n to node b and intended for sink node β . We denote the corresponding matrix forms of these variables as $\mathbf{x}(t)$, $\mathbf{v}(t)$, and $\boldsymbol{\mu}(t)$. We assume that newly injected packets $\mathbf{x}(t)$ cannot be transmit immediately. The dynamics of the information queue lengths are $\forall n \neq \beta$,

$$U_n^{c\beta}(t+1) = \max \left(U_n^{c\beta}(t) - \sum_{b \in \mathcal{N}} \mu_{nb}^{c\beta}(t), 0 \right) + \sum_{a \in \mathcal{N}} \mu_{an}^{c\beta}(t) + \mathbf{1}_{\{s_c=n\}} x_{s_c}^c(t). \quad (4.1)$$

where the indicator $1_{\{s_c=n\}} = 1$ if $s_c = n$, and 0 otherwise. At a sink node β , there is no queue for packets intended to itself, *i.e.*, $U_n^{c\beta}(t) \equiv 0$ when $n = \beta$. In practical networks, the instantaneous injecting rate and the transmission rate are all upper-bounded, which can be represented as $\max_{c,t} \sum_b v_{nb}^c(t) \leq v_{n,\max}^{out}$, $\max_{c,t} \sum_a v_{an}^c(t) \leq v_{n,\max}^{in}$ and $\max_t \sum_c x_{s_c}^c(t) \leq x_{s_c,\max}$.

In this chapter, we focus on the distributed algorithm design based on the above network coding scheme in node-capacitated networks. Readers interested in the decoding probability and implementation details can refer to [103] for more discussions.

4.3 Stochastic Network Utility Maximization Problem

We want to design an algorithm that maximizes the total utility (or social welfare) of the node-capacitated network under the stochastic network environment. One natural way to define utility is to consider the satisfaction levels of the sink nodes of all multicast sessions. However, there are practical approaches that transfer the utilities of the sink nodes to their corresponding source nodes, *e.g.*, customers pay subscription fees to their content providers. Thus we will model network utility as the satisfaction levels of the source nodes only, which are functions of their injecting rates.²

Formally, we define the utility of a source node s_c for session c as $g_{s_c}^c(\overline{x_{s_c}^c})$, which is an increasing and concave function of the source node time average in-

²The average receiving rates of sink nodes are equal to average injecting rates of source nodes, provided that the network is stable.

jecting rate

$$\overline{x_{s_c}^c} \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[x_{s_c}^{c\beta}(\tau)], \quad \forall c \in \mathcal{C}.$$

All expectations in this chapter are taken with respect to the network state $M(t)$ unless stated otherwise. Likewise, the time average link physical rate are

$$\overline{v_{nb}^c} \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[v_{nb}^c(\tau)], \quad \forall n, b \in \mathcal{N}, \forall c \in \mathcal{C}.$$

The time average link information rate are

$$\overline{\mu_{nb}^{c\beta}} \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[\mu_{nb}^{c\beta}(\tau)] \quad \forall n, b \in \mathcal{N}, \forall c \in \mathcal{C}, \forall \beta \in \mathcal{T}_c.$$

The network utility is defined as the summation of utilities of all sessions in the network, *i.e.*, $\sum_{c \in \mathcal{C}} g_{s_c}^c(\overline{x_{s_c}^c})$.

Our goal is to maximize the network utility, subject to network stability under the random network topology $S(t)$ and node capacities $J(t)$. We say that an information queue $U_n^{c\beta}(t)$ is *stable* if

$$\overline{U_n^{c\beta}} = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[U_n^{c\beta}(\tau)] < \infty,$$

and the network is *stable* if each individual queue is stable. Based on this, we want to solve the following stochastic Network Utility Maximization (NUM) problem.

$$\text{stochastic NUM: } g^* = \max_c \sum_c g_{s_c}^c(\overline{x_{s_c}^c})$$

$$\text{subject to network stability,} \quad (4.2)$$

$$\sum_{b,c} \overline{v_{nb}^c} \leq \bar{J}_n, \quad \forall n \in \mathcal{N}, \quad (4.3)$$

$$\gamma(t) \in \Gamma_{M(t)}, \quad \forall t, \quad (4.4)$$

variables $\gamma(t), \forall t$.

Constraint (4.3) means that the maximum average upload rate of node n is no larger than the parameter \bar{J}_n preset by the node n . Constraint (4.4) is an abbreviation for all instantaneous constraints in the network. The network state $\mathbf{M}(t) = (\mathbf{S}(t), \mathbf{J}(t))$, the variable $\boldsymbol{\gamma}(t) \triangleq (\mathbf{x}(t), \mathbf{v}(t), \boldsymbol{\mu}(t))$ and $\Gamma_{\mathbf{M}(t)}$ is the region characterized by the following instantaneous constraints:

$$\sum_c x_{s_c}^c(t) \leq x_{s_c, \max}, \quad \forall c, \quad (4.5)$$

$$\mu_{nb}^{c\beta}(t) \leq v_{nb}^c(t), \quad \forall n, b, c, \beta \in \mathcal{T}_c, \quad (4.6)$$

$$\sum_c v_{nb}^c(t) \leq r_{nb}(S_{nb}(t)), \quad \forall n, b, \quad (4.7)$$

$$\sum_{b,c} v_{nb}^c(t) \leq J_n(t), \quad \forall n, \quad (4.8)$$

where (4.5) means that the new injecting rate is upper-bounded at every source node. Network coding constraint (4.6) requires that the information rate of any link for any session cannot be larger than the corresponding physical rate at any time. Link capacity constraint (4.7) denotes that the physical rate of any link can not exceed its link capacity at any time. Node capacity constraint (4.8) represents that the physical rate flowing out of each node is upper-bounded by its physical upload capacity at any time.

To solve the stochastic NUM problem, let us consider a class of stationary randomized policies with the following structure. After observing the network state $\mathbf{M}(t) = \mathbf{m}$ at the beginning of a time slot t , we randomly choose one rate allocation strategy $\boldsymbol{\gamma}(t)$ from a countably collection of $\boldsymbol{\gamma}^m = \{\gamma_1^m, \gamma_2^m, \dots\}$ with probabilities $\{\alpha_k^m\}_{k=1}^\infty$, where $\sum_{k=1}^\infty \alpha_k^m = 1$. Notice that the decision is independent of time t , and thus is stationary.

Theorem 4.1. *There exists a stationary randomized policy $\boldsymbol{\gamma}^*(t) = (\mathbf{x}^*(t), \boldsymbol{\mu}^*(t), \mathbf{v}^*(t))$*

that stabilizes the network, satisfies (4.4), and have the following properties:

$$\sum_c g_n^c(E[x_{s_c}^{c*}(t)]) = g^* \quad (4.9)$$

$$E[x_n^{c*}(t)] + \sum_a E[\mu_{an}^{c\beta*}(t)] \leq \sum_b E[\mu_{nb}^{c\beta*}(t)], \forall n, c, \beta \neq n, \beta \in \mathcal{T}_c \quad (4.10)$$

$$\sum_{b,c} E[v_{nb}^{c*}(t)] \leq \bar{J}_n, \quad \forall n \quad (4.11)$$

Theorem 4.1 is a special case of Theorem 4.5 in [21]. The proof is omitted here. Theorem 4.1 states that the NUM problem can be solved by some stationary randomized policy, which satisfies the constraints (4.2)–(4.4), and achieves the maximum utility as in (4.9).

4.4 Low Complexity Distributed Algorithm

Though we can solve the stochastic NUM problem, finding the optimal stationary random policy is difficult in practice as it requires the complete information of network topologies and node upload capacities. In this section, we will design a low complexity **Incentive dynamic Distributed cross-layer Control (IDC) algorithm**, which requires no statistic information of network topology and node capacities. We will further discuss its incentive property and performance in Section 4.5.

Before presenting the algorithm, we introduce the virtual queue as in [79],

$$Y_n(t+1) = \max(Y_n(t) - \bar{J}_n, 0) + \sum_{b,c} v_{nb}^c(t), \forall n \in \mathcal{N}, \quad (4.12)$$

which transform the time-average constraint (4.3) into a single server queue with traffic rate $\sum_{b,c} v_{nb}^c(t)$ and constant serving rate \bar{J}_n . Intuitively, if virtual queue (4.12) is stable, then we have average traffic rate should be no larger than the serving rate, *i.e.*, (4.3) is satisfied.

In the sequel, we will design a dynamic and distributed pricing and resource allocation algorithm that can stabilize all information queues (4.1) and virtual queues (4.12), satisfy the instantaneous constraints (4.4), and achieve arbitrarily close to the maximum network utility through proper parameter choices.

To provide nodes incentives to relay other users' traffic, each node n charges a unit price $q_n^{c\beta}(t) = U_n^{c\beta}(t)/V$ for delivering a session c packet intended for a sink $\beta \in \mathcal{T}_c$, $\beta \neq n$. This price is proportional to the information queue length $U_n^{c\beta}(t)$, and V is a system design parameter used to trade-off optimality and delay. Moreover, each node n will maintain an index $p_n(t) \triangleq Y_n(t)/V$ to monitor the usage of its upload capacity. According to these prices and indices, nodes will perform session selection, neighbor selection (*i.e.*, link scheduling), rate allocation and network coding. The detailed algorithm are as follows:

In every slot t , source node s_c chooses the amount of data $x_{s_c}^c(t)$ to inject into the network based on its local prices $q_{s_c}^c(t) = \sum_{\beta \in \mathcal{T}_c} q_{s_c}^{c\beta}(t)$, by solving the following optimization problem.

- *Admission control:*

$$\text{Maximize}_{x_{s_c}^c(t), \forall c \in \mathcal{C}} \sum_{c \in \mathcal{C}} g_{s_c}^c(x_{s_c}^c(t)) - \sum_c q_{s_c}^c(t) x_{s_c}^c(t) \quad (4.13)$$

$$\text{Subject to } 0 \leq \sum_{c \in \mathcal{C}} x_{s_c}^c(t) \leq x_{s_c, \max}. \quad (4.14)$$

In every time slot t , each node $n \in \mathcal{N}$ chooses to perform network coding on only one session and transmit the coded packets to only one neighboring node, based on the following weights:

$$W_{nb}^{c\beta}(t) = \max(q_n^{c\beta}(t) - q_b^{c\beta}(t) - \delta_{\max}/V, 0), \forall n, b, c, \beta, \quad (4.15)$$

where $\delta_{\max} \triangleq \max\{v_{n, \max}^{out}, v_{n, \max}^{in} + 1_{\{s_c=n\}} x_{s_c, \max}\}$.

- *Select one network coding session for each neighbor b :*

$$\hat{c}_{nb}(t) = \arg \max_{c \in \mathcal{C}} \sum_{\beta \in \mathcal{T}_c} W_{nb}^{c\beta}(t), \quad \forall b \in \mathcal{N}. \quad (4.16)$$

- *Select one neighbor to transmit:*

$$b_n^*(t) = \arg \max_{\{b: S_{nb}(t)=1\}} \sum_{\beta \in \mathcal{T}_{\hat{c}_{nb}(t)}} W_{nb}^{\hat{c}_{nb}(t)\beta}(t). \quad (4.17)$$

Then the corresponding network coding session is

$$c_n^*(t) = \hat{c}_{nb_n^*(t)}(t). \quad (4.18)$$

Once node n determines next hop node $b_n^*(t)$ and the single session $c_n^*(t)$ for network coding, it needs to further compute the transmission rate and network coding scheme.

- *Rate allocation:* The transmission rate at link (n, b) for session c is

$$\mu_{nb}^{c\beta}(t) = \begin{cases} v_{nb}^c(t), & \text{if } \beta \in \mathcal{T}_c \text{ and } W_{nb}^{c\beta}(t) > 0, \\ 0, & \text{otherwise.} \end{cases} \quad (4.19)$$

where

$$v_{nb}^c(t) = \begin{cases} J_n(t), & \text{if } \sum_{\beta} W_{nb}^{c\beta} - p_n(t) > 0, c = c_n^*(t), b = b_n^*(t) \\ 0, & \text{otherwise.} \end{cases} \quad (4.20)$$

- *Network coding:* The session $c_n^*(t)$ packet physically transmitted on link $(n, b_n^*(t))$ is a random linear combination (in the predetermined coding field \mathbb{F}_q) of packets corresponding to a subset of virtual queues on $(n, b_n^*(t))$. Each virtual queue is associated with a different sink in $\mathcal{T}_{c_n^*}$. The subset of $\mathcal{T}_{c_n^*(t)}$ associated with the transmissions consists of all sinks β that satisfies $W_{nb_n^*(t)}^{c_n^*(t)\beta} > 0$.

4.5 Performance Analysis

4.5.1 Network Stability

Before prove the stability property of the IDC algorithm proposed in Section 4.3, we first introduce some notations for the purpose of performance analysis. Let $g_{\max} \triangleq \max_{c,x} g_{s_c}^c(x)$ be the maximum first derivative of utility functions, since functions $g_{s_c}^c$'s are concave increasing with $g_{s_c}^c(0) = 0$, it is easy to see that $g_{\max} = \max_{n,c} g_n^{c'}(0)$. Let $T \triangleq \max_c |\mathcal{T}_c|$ denote the maximum number of sinks of a single multicast session.

Theorem 4.2. *If the network states $M(t)$ are i.i.d. over time, then for any fixed parameter $V > 0$, all queue lengths in (4.1) and (4.12) are bounded by some constants:*

$$U_n^{c\beta}(t) \leq Vg_{\max} + \delta_{\max}, \forall t, n \in \mathcal{N}, c \in \mathcal{C}, \beta \in \mathcal{T}_c, \quad (4.21)$$

$$Y_n(t) \leq TVg_{\max} + v_{n,\max}^{out}, \quad \forall t, n \in \mathcal{N}. \quad (4.22)$$

Proof. We first prove (4.21) by induction. It is easy to see that at slot $t = 0$, no packets are in the network and $U_n^{c\beta}(0) = 0$, thus the queue length bound (4.21) obviously holds. Now suppose (4.21) holds for some slot t . We consider the queue length bound in slot $t + 1$ in the following cases:

- *Case 1:* $U_n^{c\beta}(t + 1) \leq U_n^{c\beta}(t)$, clearly the queue length bound also holds at slot $t + 1$.
- *Case 2:* $U_n^{c\beta}(t + 1) > U_n^{c\beta}(t)$ and $1_{\{s_c=n\}}x_{s_c}^c(t) > 0$, which implies that $U_n^{c\beta}(t) < Vg_{\max}$. It is because when $q_{s_c}^c(t) \geq g_{\max}$, the optimal solution for the admission control problem (4.13) is $x_{s_c}^c(t) = 0$. Since δ_{\max} represents the largest backlog changes in a slot, it follows that $U_n^{c\beta}(t + 1) \leq Vg_{\max} + \delta_{\max}$.

- *Case 3:* $U_n^{c\beta}(t+1) > U_n^{c\beta}(t)$ and $1_{\{s_c=n\}}x_{s_c}^c(t) = 0$, which implies there are packets coming to node n from other nodes. By the rate allocation (4.19) in the above algorithm, there must exist at least one node a , such that $W_{an}^{c\beta}(t) > 0$, i.e.,

$$W_{an}^{c\beta}(t) = (U_a^{c\beta}(t) - U_n^{c\beta}(t) - \delta_{\max})/V > 0. \quad (4.23)$$

Thus we further calculate that $U_n^{c\beta}(t+1) \leq U_n^{c\beta}(t) + \delta_{\max} < U_a^{c\beta}(t) \leq Vg_{\max} + \delta_{\max}$. The first inequality follows the queue dynamics in (4.1), the second inequality follows (4.23), and the third inequality follows the induction assumption.

The above analysis shows that the information queue bound (4.21) holds for all $t, n, c, \beta \in \mathcal{T}_c$.

Likewise, we can prove the virtual queue bound (4.22). Suppose that the inequality (4.22) holds for $Y_n(t)$, and consider the following two cases.

- *Case 1:* $Y_n(t+1) \leq Y_n(t)$, clearly the queue length bound (4.22) also holds at slot $t+1$.
- *Case 2:* $Y_n(t+1) > Y_n(t)$, which implies $p_n(t) < \sum_{\beta} W_{nb}^{c\beta}(t)$. This is because when $p_n(t) \geq \sum_{\beta} W_{nb}^{c\beta}(t)$, the optimal solution for rate allocation (4.20) $v_{nb}^c(t) = 0$, and the virtual queue cannot increase. Thus $Y_n(t) \leq \sum_{\beta} (U_n^{c\beta}(t) - U_b^{c\beta}(t) - \delta_{\max})$, further by the information queue bound (4.21) and the fact $U_b^{c\beta}(t) \geq 0$, we can obtain that $Y_n(t) \leq TVg_{\max}$. Then by (4.12), it follows $Y_{n+1}(t) \leq TVg_{\max} + v_{n,\max}^{out}$.

The above analysis shows that the information queue bound (4.22) holds for all time t and all node n . □

4.5.2 Network Utility Maximization

We will show the IDC algorithm can achieve arbitrarily close to the maximum network utility by a proper choice of the parameter V .

Theorem 4.3. *If the network states $\mathbf{M}(t)$ are i.i.d. over time, then the social optimal value achieved by IDC algorithm satisfies:*

$$\inf_c \sum g_{s_c}^c(\bar{x}_{s_c}^c) \geq g^* - O(1/V), \quad (4.24)$$

where g^* is the optimal value of the NUM problem.

Proof. Define the queue length vector $\Theta(t) \triangleq \{U(t), \mathbf{Y}(t)\}$, the Lyapunov function

$$L(\Theta(t)) \triangleq \frac{1}{2} \left[\sum_{n,c,\beta} (U_n^{c\beta}(t))^2 + \sum_n (Y_n(t))^2 \right],$$

and the Lyapunov drift

$$\Delta(\Theta(t)) \triangleq E[L(\Theta(t+1)) - L(\Theta(t)) | \Theta(t)], \quad (4.25)$$

By the information queue dynamic (4.1), we have

$$\begin{aligned} U_n^{c\beta}(t+1)^2 &\leq \left(U_n^{c\beta}(t) - \sum_b \mu_{nb}^{c\beta}(t) \right)^2 + \left(\sum_a \mu_{an}^{c\beta}(t) + x_{s_c}^c(t) \right)^2 \\ &\quad + 2U_n^{c\beta}(t) \left(\sum_a \mu_{an}^{c\beta}(t) + x_{s_c}^c(t) \right) \\ &= U_n^{c\beta}(t)^2 + \left(\sum_b \mu_{nb}^{c\beta}(t) \right)^2 + \left(\sum_a \mu_{an}^{c\beta}(t) + x_{s_c}^c(t) \right)^2 \\ &\quad + 2U_n^{c\beta}(t) \left(\sum_a \mu_{an}^{c\beta}(t) + x_{s_c}^c(t) - \sum_b \mu_{nb}^{c\beta}(t) \right). \end{aligned}$$

similarly for virtual queue (4.12), we have

$$Y_n(t+1)^2 \leq Y_n(t)^2 + \bar{J}_n^2 + \left(\sum_{b,c} v_{n,b}^c(t) \right)^2 + 2Y_n(t) \left(\sum_{b,c} v_{n,b}^c(t) - \bar{J}_n \right)$$

then we can calculate that

$$\begin{aligned}
 & \Delta(\Theta(t)) - V \sum_{n,c} E[g_{s_c}^c(x_{s_c}^c(t)) | \Theta(t)] \\
 & \leq \tilde{B} + \sum_{n,c,\beta} U_n^{c\beta}(t) E \left[\sum_a \mu_{an}^{c\beta}(t) + x_{s_c}^c(t) - \sum_b \mu_{nb}^{c\beta}(t) | \Theta(t) \right] \\
 & \quad + \sum_n Y_n(t) E \left[\sum_{b,c} v_{nb}^c(t) - \bar{J}_n | \Theta(t) \right] - V \sum_{n,c} E[g_{s_c}^c(x_{s_c}^c(t)) | \Theta(t)] \\
 & \leq B - \sum_{n,b,c,\beta} \delta_{max} E \left[\mu_{nb}^{c\beta}(t) | \Theta(t) \right] \\
 & \quad + V \sum_{n,c,\beta} q_n^{c\beta}(t) E \left[\sum_a \mu_{an}^{c\beta}(t) + x_{s_c}^c(t) - \sum_b \mu_{nb}^{c\beta}(t) | \Theta(t) \right] \\
 & \quad + V \sum_n p_n(t) E \left[\sum_{b,c} v_{nb}^c(t) - \bar{J}_n | \Theta(t) \right] - V \sum_{n,c} E[g_{s_c}^c(x_{s_c}^c(t)) | \Theta(t)] \quad (4.26) \\
 & = B + V \sum_c E \left\{ -g_{s_c}^c(x_{s_c}^c(t)) + x_{s_c}^c(t) \sum_\beta q_c^{c\beta}(t) | \Theta(t) \right\} \\
 & \quad - V \sum_{n,b,c,\beta} \left(q_n^{c\beta}(t) - q_b^{c\beta}(t) - \delta_{max}/V \right) E \left\{ \mu_{nb}^{c\beta}(t) | \Theta(t) \right\} \\
 & \quad + V \sum_{n,b,c} p_n(t) E \{ v_{nb}^c(t) | \Theta(t) \} - V \sum_n p_n(t) \bar{J}_n \quad (4.27)
 \end{aligned}$$

where $\tilde{B} \triangleq \frac{1}{2} \sum_n v_{n,\max}^{in\ 2} + (v_{n,\max}^{out} + x_{s_c,\max})^2 + \bar{J}_n^2 + v_{n,\max}^{out\ 2}$, and $B \triangleq \tilde{B} + \delta_{max} \sum_n v_{n,\max}^{out}$.

We can show that IDC algorithm minimizes the right hand side of (4.27) at any time t over any possible control policy of $\gamma(t) = (\mathbf{x}(t), \boldsymbol{\mu}(t), \mathbf{v}(t))$ as follows. First, the admission control in (4.13) is actually the second term. Then with canceling our all the constant, we can write the remaining parts as the following

optimization problem: $\forall n$

$$\begin{aligned}
 & \text{maximize } f(t) \triangleq & (4.28) \\
 & \sum_{c,b,\beta} (q_n^{c\beta}(t) - q_b^{c\beta}(t) - \delta_{\max}/V) \mu_{nb}^{c\beta}(t) - p_n(t) v_{nb}^c(t) \\
 & \text{subject to } v_{nb}^c(t) \geq \mu_{nb}^{c\beta}(t), \quad \forall n, b \in \mathcal{N}, c \in \mathcal{C}, \beta \in \mathcal{T}_c, \\
 & \sum_{b,c} v_{nb}^c(t) \leq c_n(t), \quad \forall n \in \mathcal{N}, \\
 & \sum_c v_{nb}^c \leq r_{nb}(S(t)), \quad \forall n, b \in \mathcal{N}. \\
 & \text{variables } \mathbf{v}(t), \boldsymbol{\mu}(t)
 \end{aligned}$$

We first maximize over $\boldsymbol{\mu}(t)$, we have $\mu_{nb}^{c\beta}(t) = v_{nb}^c(t)$, if $q_n^{c\beta}(t) - q_b^{c\beta}(t) - \delta_{\max}/V > 0$, and $\mu_{nb}^{c\beta}(t) = 0$ otherwise. Substitute this solution, we have a optimization problem with the only variable $\mathbf{v}(t)$, that is,

$$\begin{aligned}
 & \text{maximize } f(t) = \sum_{c,b} \left(\sum_{\beta} W_{nb}^{c\beta} - p_n(t) \right) v_{nb}^c(t) \\
 & \text{subject to } \sum_{b,c} v_{nb}^c(t) \leq c_n(t), \quad \forall n \in \mathcal{N}, \\
 & \sum_c v_{nb}^c \leq r_{nb}(S(t)), \quad \forall n, b \in \mathcal{N}. \\
 & \text{variables } \mathbf{v}(t)
 \end{aligned}$$

For this optimization, clearly the optimal solution is to choose the session c^* and neighbor b^* with largest weight $\sum_{\beta} W_{nb}^{c\beta}$ as (16)-(18), since p_n is some constant. The rate allocation is just (19) and (20).

Thus since IDC algorithm minimizes the right hand side of (4.27) at any time t over any possible policy $\gamma(t)$, if we replace $\gamma(t)$ in the right hand side of (26) by the stationary random policy $\gamma^*(t)$ in Theorem 4.1, then the inequality still holds. By the inequalities in Theorem 4.1 we have

$$\Delta(\Theta(t)) - V \sum_{n,c} E\{g_{s_c}^c(x_{s_c}^c(t))\} \leq B - Vg^*. \quad (4.29)$$

Since (4.29) holds for all time, we take expectation on both sides of this inequality and sum time for all $\tau = 0, 1, \dots, t - 1$, then

$$L(\Theta(t)) - L(\Theta(0)) - V \sum_{\tau=0}^{t-1} \sum_{n,c} E\{g_{s_c}^c(x_{s_c}^c(\tau))\} \leq t(B - Vg^*). \quad (4.30)$$

Dividing (4.30) by t and V , further using the concavity of $g_{s_c}^c(\cdot)$ and the fact $L(\Theta(t)) \geq 0$, we have

$$\begin{aligned} \sum_{n,c} g_{s_c}^c \left(\frac{1}{t} \sum_{\tau=0}^{t-1} E\{x_{s_c}^c(\tau)\} \right) &\geq \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n,c} E\{g_{s_c}^c(x_{s_c}^c(\tau))\} \\ &\geq g^* - \frac{B}{V} - \frac{L(\Theta(0))}{Vt} \end{aligned} \quad (4.31)$$

By taking \liminf of (4.31) as $t \rightarrow \infty$, the result of (4.24) follows. \square

By Theorem 4.3, we can achieve arbitrarily close to the optimal utility by increasing the parameter V . However, as V increases, Theorem 4.2 shows that the average queue lengths will also increase linearly in V . By Little's Law, average queue lengths are proportional to average queue delays. Thus this presents a optimality-delay trade-off of $[O(1/V), O(V)]$.

4.5.3 The Incentives Issue

In Section 4.2, we show that the source node of a multicast session has a utility function representing the satisfaction levels of all sinks of that session. What if a node is neither a source or a sink, *i.e.*, simply a helper for relaying traffic for other nodes? Does a helper have incentive to stay in the network?

The IDC algorithm proposed in Section 4.3 suggests that a helper can make non-negative profit from providing relay services, and thus has the incentive to stay in the network. Figure 4.5 illustrates how a helper node n provides the relay service according to the algorithm, which involves charging its upstream nodes

and paying its downstream nodes. Let us focus on the transmission of a single packet, *e.g.*, the gray packet in Fig. 4.5. Assumes that this packet arrives at node n in time slot $t' (< t)$, where node n charges node a a price of $q_n^{c\beta}(t')$. Then the packet leaves node n in time slot t , where node n pays node b a price of $q_n^{c\beta}(t)$. Node n will have incentive to provide the relay service if $q_n^{c\beta}(t') - q_n^{c\beta}(t) \geq 0$ for every packet. We will prove that this is indeed true in our algorithm.

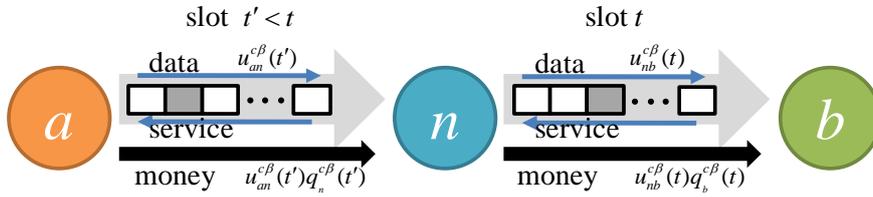


Figure 4.5: Relay service of node n

As the first step, we define the profit of node n from providing relay service from time 0 to $t - 1$ as

$$\zeta_n(t) = \sum_{\tau=0}^{t-1} \left(\sum_c 1_{\{s_c=n\}} q_{s_c}^c(t) x_{s_c}^c(t) + \sum_{c,a,\beta} q_n^{c\beta}(t) \mu_{an}^{c\beta}(t) - \sum_{c,b,\beta} q_b^{c\beta}(t) \mu_{nb}^{c\beta}(t) \right).$$

Theorem 4.4. *If the network states $\mathbf{M}(t)$ are i.i.d. over time, then for any fixed parameter $V > 0$, a node n obtains a non-negative profit for all t , i.e.,*

$$\zeta_n(t) \geq 0, \quad \forall n \in \mathcal{N}. \quad (4.32)$$

Proof. Remember that (4.28) in the proof of Theorem 4.3, we showed that the node selection, session selection, and the rate allocation of the IDC algorithm together maximize the value

$$f(t) = \sum_{c,b,\beta} (q_n^{c\beta}(t) - q_b^{c\beta}(t) - \delta_{\max}/V) \mu_{nb}^{c\beta}(t) - p_n(t) v_{nb}^c(t)$$

at any time t for each node n . It is clear that $f(t)$ is non-negative for all t from the above optimization, since 0 is a solution for the above optimization problem. By

this result, we can also rewrite the profit for node n as

$$\begin{aligned} \zeta_n(t) = & \sum_{\tau=0}^{t-1} \left(\sum_c 1_{\{s_c=n\}} q_{s_c}^c(t) x_{s_c}^c(t) + \sum_{c,a,\beta} q_n^{c\beta}(t) \mu_{an}^{c\beta}(t) + f(t) \right) \\ & - \sum_{\tau=0}^{t-1} \sum_{c,b,\beta} (q_n^{c\beta}(t) - \delta_{\max}/V) \mu_{nb}^{c\beta}(t) + p_n(t) v_{nb}^c(t) \end{aligned}$$

Since $f(t) \geq 0$ and $p_n(t) v_{nb}^c(t) \geq 0$, to prove $\zeta_n(t) \geq 0$ we just need to prove

$$\begin{aligned} \sum_{\tau=0}^{t-1} \left(\sum_c 1_{\{s_c=n\}} q_{s_c}^c(t) x_{s_c}^c(t) + \sum_{c,a,\beta} q_n^{c\beta}(t) \mu_{an}^{c\beta}(t) \right) \geq \\ \sum_{\tau=0}^{t-1} \sum_{c,b,\beta} (q_n^{c\beta}(t) - \delta_{\max}/V) \mu_{nb}^{c\beta}(t). \end{aligned} \quad (4.33)$$

Now let us prove (4.33).

We observe that in the IDC algorithm, all sample paths of backlog and utility are the same under any service order for the $U_{nb}^{c\beta}(t)$ queues, provided that queuing dynamics satisfy (4.1). Without loss of generality, we assume that the queues operate in the *Last In First Out* manner. In other words, the head of line packet in queue $U_{nb}^{c\beta}$ is the most recently arrived one. Since the node n charges a price $q_{nb}^{c\beta}(\tau) = U_{nb}^{c\beta}(\tau)/V$ to all packets in the queue, we know that the prices charged to each of the first $\mu_{nb}^{c\beta}(t)$ packets in the queue are at least $(U_{nb}^{c\beta}(t) - \mu_{nb}^{c\beta}(t))/V$. Since $\delta_{\max} \geq \mu_{nb}^{c\beta}(t)$ (where δ_{\max} is defined as the maximum change in the queue length in (4.15)), then node n 's income obtained from time 0 to time $t - 1$ is at least $\sum_{\tau=0}^{t-1} \sum_{c,b,\beta} (q_n^{c\beta}(t) - \delta_{\max}/V) \mu_{nb}^{c\beta}(t)$. Further notice that the left hand side of (4.33) represents node n 's total income from time 0 to $t - 1$. Thus, the inequality (4.33) follows, which also proves (4.32). \square

Theorem 4.4 shows that the helper has an incentive to join and stay in the system. This is especially useful in networks with multiple individual users, *e.g.*, P2P networks, as helpers can significantly increase the network performance [94].

Now let us consider the source nodes in the network. We can view each of them as a content provider who hires itself to do the relay. Will it lose money in the network? The answer is no, as each source node performs admission control to maximize its net surplus in our algorithm. Before formally proving this result in the next theorem, we define the net surplus for a source node n as a content provider from time 0 to $t - 1$ as

$$\xi_{s_c}(t) = \sum_{\tau=0}^{t-1} \sum_c \left[g_{s_c}^c \left(\frac{1}{t} \sum_{\tau=0}^{t-1} x_{s_c}^c(\tau) \right) - q_{s_c}^c(t) x_{s_c}^c(t) \right].$$

Theorem 4.5. *If the network states $M(t)$ are i.i.d. over time, then for any fixed parameter $V > 0$, a source node s_c obtains nonnegative profit as a content provider at any time t , i.e.,*

$$\xi_{s_c}(t) \geq 0, \quad \forall c \in \mathcal{C}. \quad (4.34)$$

Proof. According to the admission control in the IDC algorithm, each source node chooses the injection rate $x_{s_c}^c(t)$ to maximize the objective of (4.13) at each t . Note that the optimal objective value $\sum_c g_{s_c}^c(x_{s_c}^c) - \sum_c q_{s_c}^c x_{s_c}^c$ should be non-negative, since 0 is one feasible solution. Thus it follows

$$\sum_{\tau=0}^{t-1} \sum_c g_{s_c}^c(x_{s_c}^c(\tau)) - q_{s_c}^c x_{s_c}^c(\tau) \geq 0. \quad (4.35)$$

Further, by the concavity of functions $g_{s_c}^c$, we have

$$g_{s_c}^c \left(\frac{1}{t} \sum_{\tau=0}^{t-1} x_{s_c}^c(\tau) \right) \geq \frac{1}{t} \sum_{\tau=0}^{t-1} g_{s_c}^c(x_{s_c}^c(\tau)).$$

Substitute this result into (4.35), then inequality (4.34) follows immediately. \square

From the above analysis, we find that the node-capacitated network under the IDC algorithm provides the right incentives for all nodes. No matter what roles each node plays in the network (either as a content provider or a helper), it earns non-negative profit.

4.6 Summary

In this chapter, we consider a multicast scenario in a time-varying node-capacitated network with network coding. We design a dynamic and distributed pricing and resource allocation algorithm that can achieve arbitrarily close to the optimal network utility (at the expense of large delay) while maintaining the network stability. Moreover, we show this algorithm is incentive-compatible, *i.e.*, no matter what role a node plays in the network, the algorithm guarantees that the node has a non-negative profit. This result has practical importance for constructions of node-capacitated network with multiple individual users (*e.g.*, P2P networks), since it provides the proper incentives for individual nodes to join, stay, and contribute as relays in the network even if they have no interested contents.

Chapter 5

Conclusion

The motif of this thesis is to study pricing designs in various communication networks. We concentrate on the pricing mechanism designs that achieve good network performances. What is more, we hope to ensure the incentives of market participants, *e.g.*, the service provider's pursuit for the profit, user's requirements for individual surplus and quality of service. According to different design goals and different network scenarios, this thesis divides network pricing into four categories. Except the well-studied static optimization-oriented pricing, this thesis covers the other three categories: static profit-driven pricing, dynamic profit-driven pricing, and dynamic optimization-oriented pricing. We show the key design challenges and insights through in-dept investigations in three concrete network pricing instances. We now conclude by discussing some possible future directions as follows: extensions on static profit-driven pricing; extensions on dynamic profit-driven pricing; and extensions on dynamic optimization-oriented pricing.

5.1 Extensions on Static Profit-driven Pricing

In Chapter 2, we considered an instance of static profit-driven pricing. We proposed three profit-maximizing pricing schemes with various price differentiation levels from a single monopoly service provider's perspective. It is very natural to ask how these price differentiation schemes change when we further consider competition among several service providers. Price differentiation and competition are two opposite forces in making pricing strategies. Price differentiation relates to market segmentation and provision of multiple service classes. In doing so, a service provider can increase the profit by charging higher prices to high-end users. However, in a competitive environment, price differentiation in fact intensifies the competition, since each service class competes with other service classes offered by the same service provider as well as the counterparts offered by other service providers. Since competition always causes service providers to reduce prices and thus decrease the profit, it is possible in some cases that the service provider may make less profit by adopting a price differentiation scheme. There has been very few results about the relationship between competition and price differentiation. One result in [28] showed that competition always outweighs price differentiation for a duopoly network with Paris Metro pricing (flat-fee pricing). However, for other network scenarios (*e.g.*, competition among several service providers) with other pricing schemes (*e.g.*, the usage-based pricing schemes discussed in Chapter 2), we do not know the answer. Therefore, it is an interesting future direction to investigate the interaction between the price differentiation and competition, and quantify the tradeoff between them in designing the profit-maximizing pricing strategies.

5.2 Extensions on Dynamic Profit-driven Pricing

In Chapter 3, we considered an instance of dynamic profit-driven pricing. We solved the profit maximization problem of a cognitive mobile virtual network operator in a downlink OFDM transmission system with multiple time varying network parameters. Our solution is based on the elastic traffic model. It would be worthwhile to incorporate inelastic traffic, which usually has strict constraints on transmission rates and delays. Typical examples of inelastic traffic include real-time multimedia applications, *e.g.*, audio streaming, Video on Demand (VoD), and Voice over IP (VoIP). In the most general case, we can consider a hybrid system with both elastic and inelastic traffic, which is more realistic and practical. In addition, as mentioned in Section 3.2, the literature about competition in cognitive radio networks mainly focus on the static network scenario. It is also interesting to extend our dynamic model to incorporate competition among several network operators.

5.3 Extensions on Dynamic Optimization-oriented Pricing

In Chapter 4, we considered an instance of dynamic optimization-oriented pricing. We designed an incentive-compatible and distributed pricing scheme that can achieve arbitrarily close to the optimal network utility (at the expense of network delay) while maintaining the network stability. There are two interesting further study directions. One direction is the reduction of overhead for each node, *i.e.*, the number of queues that each node needs to maintain. Given a multicast network with C sessions and B sinks per session, each node needs to maintain $C \times B$

virtual queues. This can be a heavy burden for a node in large scale multicast networks. Thus we want to design a pricing scheme to reduce the number of queues per node while still maintaining the optimality, network stability and incentive-compatibility. The other direction is the reduction of delay. Our current scheme usually incurs a large network delay when it approaches the optimal performance. Specifically, when achieving the $O(1/V)$ close-to-optimal utility, our scheme can only guarantee that the incurred network delay is $O(V)$. Therefore, we want to design a delay-efficient pricing scheme with better trade-offs between performance and delay. Both of the above extensions are challenging and will find applications in many network problems.

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