# Dynamic Pricing in the Presence of Participation-Dependent Social Learning

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## ABSTRACT

For Internet-based services, users' quality of service (QoS) depends on not only the available resource (capacity) but also the number of users who use the resource simultaneously (e.g., congestion effect). When a new Internet-based service provider first enters the market, there can be uncertainties regarding both the capacity and congestion, and hence the uncertainty of QoS. In this paper, we consider a participation-dependent social learning over the QoS through users' online reviews, where the QoS changes with the number of review participants. We study how such a learning process affects the provider's dynamic pricing strategy. With a simple two-period model, we analyze the strategic interactions between the provider and the users, and characterize the provider's optimal two-period dynamic pricing policy. Our results show that when the capacity is small or the users' prior QoS belief is high, the provider will choose a higher introductory price in the first period (than the price in the second period). This is in sharp contrast with the common practice of setting a lower introductory price to attract users (when congestion is not an issue). Furthermore, the learning process is beneficial to the provider with a large capacity.

## CCS CONCEPTS

• **Networks** → **Network economics**; • **Computing methodologies** → *Machine learning approaches*; Stochastic games;

## KEYWORDS

Social learning, participation, dynamic pricing, stochastic game

This work is supported in part by the General Research Funds (Project Numbers CUHK 14219016 and CityU 11527316) established under the University Grant Committee of the Hong Kong Special Administrative Region, China, in part by US Army Research Office (ARO) Grant W911NF-16-1-0485, and in part by US Army Research Laboratory (ARL) Cooperative Agreement W911NF-17-2-0196.


*Mobihoc '18, June 26–29, 2018, Los Angeles, CA, USA*

ACM ISBN 978-1-4503-5770-8/18/06…$15.00
https://doi.org/10.1145/3209582.3209593

**ACM Reference Format:**
Qian Ma, Biying Shou, Jianwei Huang, and Tamer Başar. 2018. Dynamic Pricing in the Presence of Participation-Dependent Social Learning. In *Mobihoc '18: The Eighteenth ACM International Symposium on Mobile Ad Hoc Networking and Computing, June 26–29, 2018, Los Angeles, CA, USA.* ACM, New York, NY, USA, Article 4, 10 pages. https://doi.org/10.1145/3209582.3209593

# 1 INTRODUCTION

## 1.1 Background and Motivations

The fast development of Internet and telecommunication technologies allows the users to enjoy a variety of Internet-based (online) services. Examples of popular online services include virtual private servers (e.g., Alibaba Cloud and AlphaRacks), courses offered by online platforms (e.g., Coursera), online games (e.g., World of Warcraft), and wireless services provided by mobile virtual network operators (e.g., Google's Project Fi).

An often observed feature of the online services is that the provider's profit and users' subscription decisions depend on both the pricing policy and the quality of service (QoS). The pricing decisions are usually specified for a period of time, e.g., Alibaba Cloud charges $4.5 per month [11], Coursera charges $39 per month [12], World of Warcraft charges $14.99 per month [3], and Google's Project Fi charges $20 per month [17]. The QoS depends on many different aspects in different services, e.g., the virtual private server's resource shortages and performance degradation caused by overselling [2], the online course's lack of feedback (or interaction) and low teacher-to-student ratios due to huge numbers of enrollment [5], online games' delay jitters or packet loss due to a larger number of simultaneous players [7], and wireless services' call delay or data transmission rate due to network congestion [18]. Typically the provider chooses to publicly announce the best possible QoS to users due to publicity concerns, although the actual QoS may significantly deviate from the announced values. For example, the provider of virtual private servers may claim that the speed is 100Mbps, but the actual speed may be only 10Mbps on the average due to overselling.

To summarize, the actual QoS of an online service often depends on not only the amount of available resources (capacity) but also the number of users who subscribe to the provider and use the service simultaneously (congestion). When a provider first enters the market, its resource capacity is often unknown to users, and sometimes is even uncertain to the provider itself. For example, the network resources of Google's Project Fi are leased from several mobile network operators, including Sprint, T-Mobile, and U.S. Cellular. As a result, Google may not know its resource capacity very accurately at the beginning, as it depends on how those mobile network operators manage the resource sharing between their own customers and Fi's customers [17]. In the meantime, network congestion arises when a relatively high number of users access the service simultaneously. For example, a virtual private server's QoS, i.e., processor time, RAM, and disk space, decreases as the workload of other executing ones increases [34]. As users' subscription decisions depend on the provider's pricing policy, the QoS also varies with the pricing policy rather than being a fixed value. Consequently, the QoS experienced by the users is uncertain.

One way to resolve such uncertainty is to let early-adopters leave reviews regarding their experience with QoS in an online platform. For example, on Alibaba Cloud's forum [10], there were both positive and negative reviews on the network speed, e.g., reaching >1Mbps speed or reaching <20Kbps speed. Future users can learn the QoS from these reviews and make more informed subscription decisions, which may in turn reduce the possibility of negative service experiences (due to users' self-selection and adjustment of expectations). The provider can also benefit from user reviews by making more accurate prediction on users' future demand [29].

The mutual dependence of the QoS and users' subscription decisions leads to a new social learning problem: rather than learning over a fixed value, the object of social learning here varies with the number of *participants who leave reviews*. We call such a new learning phenomenon the *participation-dependent social learning*. In this paper, we will study how such a new form of learning affects the information generated by the learning, the service subscriptions, and the provider's profit. Our proposed participation-dependent social learning is applicable to general online services whose QoS depends on both the resource capacity and the congestion, and the uncertain QoS can be revealed through subscribers' reviews.

## 1.2 Model and Key Questions

We consider a provider offering the online service to a fixed population of users over two time periods. As commonly used in the social learning literature [29, 38] and widely adopted in Internet-based services [1], the two periods are abstract representations of the *early promotion* period and *regular sales* period of a new product or service.[1] The QoS of the service is uncertain to users, and depends on both the resource capacity and the congestion effect. We consider a total of four practical information scenarios, depending on whether the provider knows the capacity and whether users are aware of the congestion effect prior to their subscription decisions. For example, Alibaba Cloud knows its resource capacity available to its users of virtual private servers, but Google's Project Fi may not know its capacity since its resources are leased from several mobile network operators. Users who are educated or experienced are aware of the congestion effect, while others may not.

[1] Analyzing a model of more than two periods is much more complicated with potential similar insights. Hence we will consider it in the future work.

Accordingly, we analyze the game-theoretic interactions between the provider and users in the two-period model for each information scenario. At the beginning of the first period, the provider decides the first-period introductory price, and users who have nonnegative expected payoffs will choose to subscribe to the provider. At the end of the first period, all the first-period subscribers will leave reviews regarding their experienced QoS. At the beginning of the second period, the provider decides the second-period regular price. Users update their belief over the QoS based on the first-period reviews, and make their subscription decisions.

The provider's goal is to design a dynamic pricing policy to maximize its expected total profit over both periods. The price will affect users' subscription decisions, users' experienced QoS, and the information that will be revealed through the reviews. For example, let us consider the first period. If the provider adopts a high introductory price, then only a few users will subscribe, and the realized QoS is high due to small congestion effect. Meanwhile, the number of reviews will be small, and the information obtained through learning will be limited. Depending on users' initial belief of the QoS, this may discourage or encourage users' subscriptions in the second period. Hence, the provider needs to carefully choose its two-period pricing policy to control the QoS revelation and subscription dynamics. Furthermore, the provider's pricing decision is complicated by the stochasticity the presence of the participation-dependent social learning process brings.

In this paper, our goal is to answer the following three key questions:

- *How does participation-dependent social learning affect users' subscription decisions?*
- *How does participation-dependent social learning affect the provider's dynamic pricing policy?*
- *How do the provider's knowledge of capacity and users' awareness of congestion affect the provider's profit and users' payoffs?*

## 1.3 Contributions

We summarize our key contributions as follows.

- *Participation-Dependent Social Learning:* To the best of our knowledge, this is the first work that studies the impact of participation-dependent social learning on the strategic interactions between the provider and the users.
- *Dynamic Pricing Policy:* We derive the provider's optimal dynamic pricing policy that maximizes its expected total profit. The problem is challenging since participation-dependent social learning brings in stochasticity to the provider's profit maximization.
- *Two-Period Stochastic Game:* We model the interactions between the provider and the users as a two-period stochastic game, and characterize the equilibrium systematically. Our analysis captures four different practical information scenarios, depending on whether the provider knows the capacity and whether users are aware of the congestion effect.
- *Practical Insights:* Our analysis helps understand how the participation-dependent social learning affects users' subscription decisions, which facilitates the provider to optimize its profit in different scenarios. Our key insights are: (i) the

provider will set a high introductory price when the capacity is small or users' prior belief over the QoS is high; (ii) the participation-dependent social learning is beneficial to the provider who has a large capacity; and (iii) both the provider and the users will benefit from users' awareness of the congestion effect.

The rest of the paper is organized as follows. In Section 2, we review related literature. In Section 3, we present the system model. In Section 4, we derive the optimal pricing policy for congestion-unaware users. In Section 5, we derive the optimal pricing policy for congestion-aware users. Finally, we conclude in Section 6. **Due to the space limitation, most of the proofs are not included here, but can be found in the online technical report [26].**

# 2 RELATED LITERATURE

## 2.1 QoS-Sensitive Internet-Based Services

Unlike physical commodities, the QoS of Internet-based services or products can be influenced by a negative congestion effect[2], i.e., the more users use the services, the worse QoS they experience. Ma in [27] studied usage-based pricing for congestion-prone network services such as mobile, cloud or broadband services. Raja *et al.* in [31] analyzed a pricing game of servers whose QoS is unknown to users in the Internet marketplace. There have also been studies for many specific Internet-based services. For example, authors in [13, 32, 41] studied different economic models and pricing policies for cloud services. Authors in [5, 6, 19] studied user behaviors, efficiency optimization, and participation incentive mechanism design for online courses. Authors in [7, 22] analyzed the impact of the QoS, i.e., network delay and network loss, on user subscriptions to online games. Authors in [16, 18, 39, 42] studied the pricing schemes for mobile virtual network operators. None of the above works considered the uncertainty of QoS and how the participation-dependent social learning can help resolve this uncertainty.

## 2.2 Social Learning

There is considerable recent literature on the pricing design considering social learning. Yu, Debo, and Kapuscinski in [38] studied the dynamic pricing policy for strategic users considering social learning. Papanastasiou and Savva in [29] further compared the dynamic pricing policy and preannounced pricing policy. Bose *et al.* in [4] studied the monopolist dynamic pricing policy to control the information reveal. Ifrach, Maglaras, and Scarsini in [21] analyzed when the seller benefits from social learning.

Social learning has also been used recently in analyzing various Internet-based services. For example, Chen *et al.* in [8] studied the wisdom of crowds for secondary users to make more intelligent spectrum access decisions. Liu and Liu in [25] studied the labeler selection problem for crowd-sensing through online learning. Liu and Zhao in [24] studied the distributed learning problem for dynamic spectrum access. Huang, Liu, and Hao in [20] proposed two online learning methods for stochastic network optimization. Xie, Ma, and Lui in [35] studied the optimal price discount scheme to attract buyers and accelerate sellers' reputation enhancement in E-commerce system. Zhang, Cui, and Wang in [40] studied the trust evaluation by mining text feedback comments in E-commerce system. Le, Subramanian, and Berry in [23] studied the impact of imperfect reviews on social learning.

The social learning in all the above works is applicable to learning over a fixed value. In this work, we focus on participation-dependent social learning, which involves learning over a value that varies with the number of review participants. The analysis is challenging since there is mutual dependence among the pricing policy, users' subscription decisions, users' experienced QoS, and the information that will be revealed through the reviews.

[2]Some network services also benefit from the positive network effective. For example, the experiences of using a social network will improve as more users join the network. We will consider the interactions between the negative congestion effect and positive network effect in the future work.

# 3 SYSTEM MODEL

First, we introduce the provider and users, focusing on the users' QoS belief update. Then we discuss the two-period game played between the provider and users. Finally we describe four different information scenarios to be analyzed in Sections 4 and 5.

## 3.1 Provider and Users

We consider a provider (e.g., AlphaRacks, Coursera, World of Warcraft, and Google's Project Fi), providing online services to users. As in many prior work (e.g., [1, 29, 38]), we analyze a two-period model. The provider chooses its pricing policy $\boldsymbol{p} = (p_t : t = 1, 2)$, i.e., the subscription fees charged to users in two periods, to maximize its expected total profit. The first period corresponds to an early-launch period where users have little information of the QoS provided by the provider. The second-period corresponds to a regular service period where users have learnt some information of the QoS through first-period reviews. The QoS depends on the resource capacity $C$ and the number of users who use the service simultaneously.[3] The QoS is ex ante[4] uncertain, since the capacity $C$ is initially unknown to users, and the number of subscribers is to be determined by the belief of the QoS and the pricing policy.

The market consists of a large number of users, hence the impact of each user's behavior is negligible. We normalize the total mass of users to one [29]. Users are heterogeneous in their preferences over the QoS. We let $a_i$ denote user $i$'s preference over QoS, which is uniformly distributed over $[0, 1]$ [29]. We let $v_{it}$ denote user $i$'s payoff obtained by subscribing to the provider in time period $t$:

$$v_{it} = a_i \cdot q_i(C, n_t(\boldsymbol{p})) - p_t, \tag{1}$$

where $q_i(C, n_t(\boldsymbol{p}))$ is the QoS experienced by user $i$ and $n_t(\boldsymbol{p})$ is the fraction of users who subscribe to the provider in time period $t$. When there is no confusion, we write $n_t(\boldsymbol{p})$ as $n_t$.

## 3.2 Users' QoS Belief Update

Next we discuss users' belief update of the QoS.

The term $q_i(C, n_t(\boldsymbol{p}))$ in (1) is user $i$'s ex post[5] QoS, which depends on the capacity $C$ and the number of users who use the service simultaneously in time period $t$. Note that $q_i(C, n_t(\boldsymbol{p}))$ is

[3]We assume that the capacity $C$ is fixed since expanding capacity (e.g., adding more servers, instructors, or spectrum) can be very costly and could go beyond the provider's investment budget. We will consider the joint investment and pricing problem in the future work.

[4]"Ex ante" means "before the event", i.e., before users experience the service.

[5]"Ex post" means "after the event", i.e., after users experience the service.

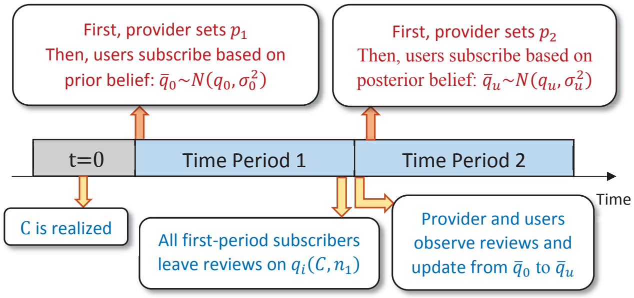


**Figure 1: Two-period game**

ex ante unknown, as a user $i$ knows the value of $q_i(C, n_t(\boldsymbol{p}))$ only after he subscribes to the provider in time period $t$. We assume that users' ex post QoS perceptions follow a normal distribution as follows,

$$q_i(C, n_t) \sim N\left(\hat{q}(C, n_t), \sigma^2\right), \tag{2}$$

where the variance $\sigma^2$ captures the degree of heterogeneity in users' QoS perceptions after subscription, due to various factors such as channel fading in wireless services offered by Google's Project Fi. We assume that $\sigma^2$ is publicly known by the provider and users. The term $\hat{q}(C, n_t)$ in (2) is users' mean QoS. In this paper, $\hat{q}(C, n_t)$ follows the following form:

$$\hat{q}(C, n_t) = \frac{C}{n_t}, \tag{3}$$

which is a function that increases in the capacity and decreases in the number of subscribers.[6] The formulation in (3) is widely used in cloud services [13, 28], online courses [5], online games [30], and wireless services [9, 42].

The mean QoS experienced by the users who have participated in the first period, $\hat{q}(C, n_1)$, is the object of social learning through reviews. We assume that users have a common and public (known to the provider) prior belief over $\hat{q}(C, n_1)$ at the beginning of the first period, which follows a normal distribution:

$$\bar{q}_0 \sim N\left(q_0, \sigma_0^2\right), \tag{4}$$

with the mean $q_0 > 0$. The variance $\sigma_0^2$ reflects users' knowledge of the impact of various factors such as channel fading in wireless services offered by Google's Project Fi.

Users who subscribe to the provider in the first period leave reviews on their experiences of QoS. We assume that the first-period subscribers leave reviews that truthfully reflect their experienced QoS.[7] At the beginning of the second period, all users observe the first-period reviews, and update their common belief[8] over the mean QoS from $\bar{q}_0$ to $\bar{q}_u$ according to a Bayesian learning rule [14] as explained in Section 4.1.

[6]There can be other more general formulations for $\hat{q}(C, n_t)$. For example, the QoS can be a constant when $n_t$ is below a threshold and decreases in $n_t$ when above that threshold. Such a function is non-smooth and non-convex, and we will study it in the future work.
[7]In future work, we will further consider the issue of imperfect reviews [23], manipulated reviews [36, 37], and incentive mechanism design for truthful reviews [33].
[8]As the randomness of QoS depends on the interactions among users in the entire population, a user who participates in the first period will evaluate the QoS based on all reviews rather than just his own experience.

**Table 1: Information Scenarios**

| Information | Informed provider | Uninformed provider |
|---|---|---|
| **Congestion-unaware users** | Section 4.1 | Section 4.2 |
| **Congestion-aware users** | Section 5.1 | Section 5.2 |

## 3.3 Two-Period Game

Fig. 1 illustrates the two-period game between the provider and users. Before the game, at time $t = 0$, the resource capacity $C$ is determined by the provider's investment and purchasing decision, which is assumed to be given in our model. At time $t = 1$, the provider announces the first-period price $p_1$. Then, users observe $p_1$ and make their subscription decisions. We denote $n_1 \in [0, 1]$ as the fraction of users who subscribe to the provider in the first period. Every first-period subscriber experiences (possibly different) online services and leaves the review of his experienced QoS at the end of the first period. At time $t = 2$, the provider and users observe the reviews and update their belief over the QoS. The provider announces the second-period price $p_2$, and users make their second-period subscription decisions. We will introduce the game in detail in Section 4.1.

## 3.4 Information Scenarios

We consider a total of four information scenarios, depending on what information is known by the provider and users.

We consider two possibilities for the provider: (i) *Informed provider:* it knows the capacity $C$ at $t = 1$ (e.g., Alibaba Cloud knows its resource capacity available to its users of virtual private servers), and (ii) *Uninformed provider:* it does not know the capacity $C$ at $t = 1$ (e.g., Google's Project Fi may not know its capacity since its resources are leased from several mobile network operators).

Users do not know the capacity at $t = 1$. There are also two possibilities for users: (i) *Congestion-unaware users:* they make their subscription decisions in both periods based only on the QoS belief and they are not aware of the fact that the QoS of online services is influenced by the congestion effect, that is, they do not know the QoS relationship in (3), and (ii) *Congestion-aware users:* they are educated or experienced enough to know the relationship in (3), and make their subscription decisions in the second period considering the congestion effect.

Table 1 summarizes these four different information scenarios. Table 2 lists some key notations in this paper.

# 4 DYNAMIC PRICING FOR CONGESTION-UNAWARE USERS

In this section, we discuss the two-period game for congestion-unaware users, i.e., users do not know the dependence of QoS on the congestion effect. We first analyze the informed provider scenario, and then analyze the uninformed provider scenario.

## 4.1 Informed Provider with Congestion-Unaware Users

The provider and users play a two-period stochastic game as shown in Fig. 1. We analyze the game through backward induction, and

**Table 2: Key Notations**

| Symbol | Physical Meaning |
|---|---|
| $\boldsymbol{p} = (p_1, p_2)$ | The provider's two-period pricing policy |
| $C$ | The provider's resource capacity |
| $n_t(\boldsymbol{p})$ | The fraction of subscribers in time period $t$ |
| $q_i(C, n_t(\boldsymbol{p}))$ | The QoS to user $i$ in time period $t$ |
| $\hat{q}(C, n_t(\boldsymbol{p}))$ | The unobservable mean QoS in time period $t$ |
| $\sigma^2$ | The public heterogeneity of QoS perceptions |
| $\bar{q}_0$ | The public prior belief over $\hat{q}(C, n_1)$ |
| $q_0$ | The public mean of the prior belief |
| $\sigma_0^2$ | The public variance of the prior belief |
| $R(n_1)$ | The average rating of the first-period reviews |
| $\bar{q}_u$ | The posterior belief over $\hat{q}(C, n_1)$ |
| $q_u$ | The mean of the posterior belief |
| $\sigma_u^2$ | The variance of the posterior belief |
| $\gamma = \sigma_0^2/\sigma^2$ | The public social learning influence parameter |
| $a_i$ | User $i$'s preference over QoS |
| $v_{it}$ | The payoff of user $i$ in time period $t$ |
| $\pi$ | The provider's expected total profit |

examine the impact of system parameters on the provider's optimal pricing policy.

*4.1.1 Second-Period Subgame.* We first characterize users' QoS belief at the beginning of the second period.

LEMMA 4.1. *Assume $n_1$ users subscribe to the provider in the first period and leave reviews with an average QoS rating of $R(n_1)$. Then the posterior belief of the QoS (at the beginning of the second period), $\bar{q}_u$, is normally distributed,*

$$\bar{q}_u \sim N\left(q_u, \sigma_u^2\right), \tag{5}$$

*with the mean*

$$q_u = \frac{1}{n_1\gamma + 1} q_0 + \frac{n_1\gamma}{n_1\gamma + 1} R(n_1), \text{ where } \gamma = \frac{\sigma_0^2}{\sigma^2}, \tag{6}$$

*and the variance*

$$\sigma_u^2 = \frac{\sigma_0^2}{n_1\gamma + 1}. \tag{7}$$

PROOF. See Appendix A in the technical report [26]. □

Lemma 4.1 captures the tradeoff between review accuracy and QoS degradation of the first period. A larger $n_1$ implies that the average review rating $R(n_1)$ is more credible, hence the weight $n_1\gamma/(n_1\gamma + 1)$ of $R(n_1)$ in (6) is increasing in $n_1$. On the other hand, $R(n_1)$ itself decreases in $n_1$, as more subscribers lead to more congestion and a lower realized QoS.

The ratio $\gamma$ in (6) can be interpreted as the *social learning influence parameter.* When $\gamma$ goes to zero, the social learning process is inactive (i.e., $\bar{q}_u = \bar{q}_0$), either because there is no ex ante uncertainty ($\sigma_0 \to 0$) or because reviews reveal no useful information ($\sigma \to +\infty$). When $\gamma \to \infty$, the social learning process dominates the posterior belief (i.e., $\bar{q}_u \sim N(R(n_1), 0)$), as users will completely abandon their prior belief $\bar{q}_0$ and entirely trust the social learning result $R(n_1)$.

Note that *users' belief update described in Lemma 4.1 is applicable to both congestion-aware users and congestion-unaware users*, since the users' behaviors in both information scenarios are the same in the first period.[9]

With the QoS belief characterized in Lemma 4.1, a user $i$'s expected payoff of subscribing in the second period is

$$E[v_{i2}] = a_i E[\bar{q}_u] - p_2 = a_i q_u - p_2. \tag{8}$$

We assume that users are risk-neutral and want to maximize their expected payoffs. A user $i$ will subscribe to the provider if and only if $q_u > 0$ and $a_i \geq \frac{p_2}{q_u}$. Hence, given that $q_u > 0$, the fraction of second-period subscribers is

$$n_2 = \max\left\{0, 1 - \frac{p_2}{q_u}\right\}. \tag{9}$$

The provider's second-period profit is

$$\pi_2(p_2) = p_2 \cdot n_2 = p_2 \cdot \max\left\{0, 1 - \frac{p_2}{q_u}\right\}. \tag{10}$$

We derive the unique equilibrium of the second-period subgame as follows.

LEMMA 4.2. *Given any posterior mean QoS belief $q_u$, there exists a unique equilibrium in the second-period subgame played between the informed provider and congestion-unaware users. Specifically:*

- *The provider's optimal second-period pricing policy is*

$$p_2^*(q_u) = \begin{cases} \frac{q_u}{2}, & \text{if } q_u > 0, \\ 0, & \text{if } q_u \leq 0. \end{cases} \tag{11}$$

- *When $q_u > 0$, all users with $a_i \in [1/2, 1]$ will subscribe to the provider. When $q_u \leq 0$, no user will subscribe to the provider.*
- *The provider's optimal second-period profit is*

$$\pi_2^*(p_2^*; q_u) = \begin{cases} \frac{q_u}{4}, & \text{if } q_u > 0, \\ 0, & \text{if } q_u \leq 0. \end{cases} \tag{12}$$

PROOF. See Appendix B in the technical report [26]. □

Lemma 4.2 shows that the provider's second-period profit maximization depends on $q_u$, which relates to the first-period subscription fraction $n_1$ as in Lemma 4.1. When $n_1$ is larger, more information of QoS is revealed through reviews, but the average review rating $R(n_1)$ will be smaller. In the following, we derive $n_1$ in the first period.

*4.1.2 First-Period Subgame.* In the first period, user $i$'s expected payoff by subscribing to the provider is:

$$E[v_{i1}] = a_i E[\bar{q}_0] - p_1 = a_i q_0 - p_1. \tag{13}$$

User $i$ will subscribe to the provider if $E[v_{i1}] \geq 0$. Hence we have:

LEMMA 4.3. *Given any first-period price $p_1$, a user $i$ subscribes in the first period if and only if $a_i \geq \frac{p_1}{q_0}$.*

[9] Even the congestion-aware users are not able to estimate the congestion in the first period, due to the lack of knowledge of capacity $C$.

Lemma 4.3 applies to all four information scenarios listed in Table 1.

Based on Lemma 4.3, we can calculate the fraction of users subscribing in the first period:

$$n_1 = \max\left\{0, 1 - \frac{p_1}{q_0}\right\}. \tag{14}$$

The remaining analysis is to determine the provider's optimal first-period price $p_1$, which should maximize the provider's expected total profit in both periods. The provider needs to anticipate the impact of $p_1$ on the second period subscription. Such impact is carried over through the posterior mean QoS $q_u$, as shown in Lemma 4.2. However, the realization of the posterior mean QoS $q_u$ is ex ante uncertain, due to the noise of user reviews in (2). In other words, the provider cannot calculate the exact value of $q_u$ when it makes the decision of $p_1$. Hence, the second-period subgame equilibrium characterized in Lemma 4.2 involves some uncertainty (regarding $q_u$) at the beginning of the first period. To resolve this issue, the provider needs to form a *rational belief* over the posterior mean QoS $q_u$, which we call the "preposterior"[10] distribution of $q_u$. Such a belief depends on the information structure of the provider.[11]

Lemma 4.4. *From an informed provider's point of view, the users' posterior mean QoS belief $q_u$ follows a preposterior equilibrium normal distribution $N(q_{\mathrm{I}}, \sigma_{\mathrm{I}}^2)$, where the mean*[12]

$$q_{\mathrm{I}} = \frac{1}{n_1\gamma + 1} q_0 + \frac{n_1\gamma}{n_1\gamma + 1} \cdot \frac{C}{n_1} \tag{15}$$

*and the variance*

$$\sigma_{\mathrm{I}}^2 = \left(\frac{n_1\gamma}{n_1\gamma + 1}\right)^2 \cdot \frac{\sigma^2}{n_1}. \tag{16}$$

*For the purpose of later analysis, we also write the probability density function of this distribution as $f_{\mathrm{I}}(q_u; n_1)$, to emphasize the dependence on the first-period subscribers $n_1$.*

Proof. See Appendix C in the technical report [26]. □

Equation (15) in Lemma 4.4 reveals the relationship between the provider's knowledge of capacity $C$ (as the provider is informed) and its prediction of users' QoS belief update. Intuitively, if the provider has a larger capacity $C$, it tends to encourage more users to subscribe in the first period as to influence the belief update based on the weight of $n_1\gamma/(n_1\gamma + 1)$; meanwhile, doing so will lead to a lower QoS review $(C/n_1)$ due to the congestion.

The variance of the preposterior distribution in (16) corresponds to the degree of the departure of the posterior mean $q_u$ from the prior mean $q_0$. The variance depends on the information revealed through user reviews, and includes uncertainty of both the QoS and the noise in the first-period reviews. When the social learning influence parameter $\gamma < 1$, i.e., $\sigma_0^2 < \sigma^2$, $\sigma_{\mathrm{I}}^2$ increases with $n_1$, i.e., the posterior mean $q_u$ departs further from the prior mean $q_0$ with more reviews.

[10] "Preposterior" refers to the provider's belief regarding the users' posterior mean QoS belief $q_u$ at the time when the provider makes the decision of $p_1$.

[11] The rational belief does not depend on whether users are aware of the congestion effect or not.

[12] The subscript "I" stands for "informed".

In the first period, taking into account the users' response to $p_1$ in Lemma 4.3, as well as the probabilistic equilibrium of the second-period subgame in Lemmas 4.2 and 4.4, the provider chooses $p_1^*$ to maximize its expected total profit in two periods:

$$\pi(p_1) = p_1 \cdot \max\left\{0, 1 - \frac{p_1}{q_0}\right\} + \int_0^\infty \frac{q_u}{4} f_{\mathrm{I}}(q_u; n_1) dq_u. \tag{17}$$

Here $q_u/4$ is the optimal second-period profit as in Lemma 4.2, and $f_{\mathrm{I}}(q_u; n_1)$ is the density function described in Lemma 4.4.

As the integral of normal distribution cannot be expressed in terms of elementary functions, we cannot obtain a closed-form solution for (17). In Algorithm 1, we describe the dynamic pricing mechanism that computes the equilibrium two-period pricing policy for the informed provider and congestion-unaware users.

**Algorithm 1:** Dynamic Pricing Mechanism for the Informed Provider and Congestion-Unaware Users

**Input**: $q_0, \sigma_0^2, \sigma^2$
**Output**: $\boldsymbol{p}^* = (p_1^*, p_2^*)$
Calculate $\gamma \leftarrow \sigma_0^2/\sigma^2$;
Calculate $f_{\mathrm{I}}(q_u; n_1)$ which follows $N(q_{\mathrm{I}}, \sigma_{\mathrm{I}}^2)$ with $q_{\mathrm{I}} \leftarrow \frac{1}{n_1\gamma+1} q_0 + \frac{n_1\gamma}{n_1\gamma+1} \cdot \frac{C}{n_1}$ and $\sigma_{\mathrm{I}}^2 \leftarrow \left(\frac{n_1\gamma}{n_1\gamma+1}\right)^2 \cdot \frac{\sigma^2}{n_1}$ where $n_1 \leftarrow \max\left\{0, 1 - \frac{p_1}{q_0}\right\}$;
$p_1^* \leftarrow \arg\max_{p_1} p_1 \cdot \max\left\{0, 1 - \frac{p_1}{q_0}\right\} + \int_0^\infty \frac{q_u}{4} f_{\mathrm{I}}(q_u; n_1) dq_u$;
$n_1^* \leftarrow \max\left\{0, 1 - p_1^*/q_0\right\}$;
Update belief over QoS $\tilde{q}_u \sim N\left(q_u, \sigma_u^2\right)$ with $q_u \leftarrow \frac{1}{n_1^*\gamma+1} q_0 + \frac{n_1^*\gamma}{n_1^*\gamma+1} R(n_1^*)$ and $\sigma_u^2 \leftarrow \frac{\sigma_0^2}{n_1^*\gamma+1}$;
**if** $q_u > 0$ **then**
  $p_2^* \leftarrow q_u/2$;
**else**
  $p_2^* \leftarrow 0$;
**end**

In Theorem 4.5, we summarize several properties of the optimal pricing policy $(p_1^*, p_2^*)$.

Theorem 4.5. *In the presence of the participation-dependent social learning, there exits a unique equilibrium in the two-period stochastic game played between the informed provider and congestion-unaware users. More specifically,*

(a) *In the first period, $p_1^* < q_0$, i.e., it is never optimal for the provider to choose a first-period price that results in no subscription in the first period.*
(b) *In the second period, the provider chooses $p_2^*$ to be*

$$p_2^*(q_u) = \begin{cases} \frac{q_u}{2}, & \textit{if } q_u > 0, \\ 0, & \textit{if } q_u \le 0. \end{cases}$$

Proof. See Appendix D in the technical report [26]. □

Note that the optimal second-period price $p_2^*$ described in Theorem 4.5 is ex ante stochastic, since it depends on the posterior mean QoS $q_u$, which is uncertain before the first-period reviews

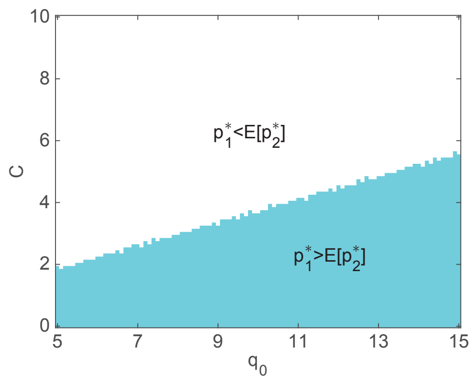


**Figure 2: Structure of the optimal pricing policy**

are realized. In the following, we explore the relationship between $p_1^*$ and the expected optimal second-period price $E[p_2^*]$ through numerical studies, and examine the impact of parameters, i.e., $C$, $q_0$ and $\gamma$, on the optimal pricing strategy and optimal expected total profit.

*4.1.3 Impact of Participation-Dependent Social Learning on the Optimal Pricing Policy.* We first numerically explore the structure of the informed provider's optimal pricing policy $(p_1^*, p_2^*)$ under different values of $C$ and $q_0$.

Fig. 2 shows the structure of the optimal pricing policy. We let $\sigma_0 = 1$ and $\sigma = 1$, i.e., the participation-dependent social learning influence parameter $\gamma = \sigma_0^2/\sigma^2 = 1$. We can see that when the capacity $C$ is small or the prior mean belief $q_0$ is large, the provider will choose $p_1^* > E[p_2^*]$ (i.e., the blue region). The reason is that when $C$ is small, a high introductory price $p_1^*$ will lead to a high QoS review rating (due to a low first-period subscription), which will attract more second-period subscribers. When $q_0$ is large, users are optimistic about the QoS, and are willing to subscribe to the provider even if the introductory price is high.

Fig. 3 examines the impact of $\gamma$ on the informed provider's optimal pricing policy. Here we choose $q_0 = 10$ and $\sigma^2 = 1$. The result highly depends on the capacity $C$. First, when the capacity is small ($C = 2$), $p_1^*$ increases with $\gamma$ and $E[p_2^*]$ decreases with $\gamma$. The reason is that as $\gamma$ becomes larger, the participation-dependent social learning process reveals more accurate information regarding the small capacity. Hence the provider chooses a higher first-period price to reduce the negative impact of social learning, and chooses a lower expected second-period price to maximize the expected total profit. Second, when the capacity is large ($C = 10$), both $\boldsymbol{p}_1^*$ and $E[p_2^*]$ increases with $\gamma$. This is because as $\gamma$ increases, user reviews reveal more accurate information about the provider's large capacity, hence the provider chooses higher prices to extract more profit from the subscribers.

## 4.2 Uninformed Provider with Congestion-Unaware Users

Now we analyze the uninformed provider scenario, where the provider does not know the capacity at the beginning of the first period.

The analysis of the second period is the same as Section 4.1, as summarized in Lemmas 4.1 and 4.2. For the first period, the analysis of users' subscription is also the same as Section 4.1, as summarized in Lemma 4.3. The key difference with Section 4.1 is the provider's pricing decision in the first period, since the uninformed provider has a different rational belief over social learning, which can be described as follows.

Lemma 4.6. *From an uninformed provider's point of view, the users' posterior mean QoS belief $q_u$ follows a preposterior equilibrium normal distribution $N(q_{\text{NI}}, \sigma_{\text{NI}}^2)$, where the mean*[13]

$$q_{\text{NI}} = q_0 \tag{18}$$

*and the variance*

$$\sigma_{\text{NI}}^2 = \left(\frac{n_1\gamma}{n_1\gamma + 1}\right)^2 \cdot \left(\frac{\sigma^2}{n} + \sigma_0^2\right) = \frac{n_1\gamma}{n_1\gamma + 1}\sigma_0^2. \tag{19}$$

*For the purpose of later analysis, we also write the probability density function of this distribution as $f_{\text{NI}}(q_u; n_1)$, to emphasize the dependence on the first-period subscribers $n_1$.*

Proof. See Appendix E in the technical report [26]. □

Different from Lemma 4.4, the uninformed provider's preposterior mean QoS belief is always $q_0$, which is ex ante not affected by the first-period reviews. The variance of the uninformed provider's preposterior is larger than that of the informed provider in Lemma 4.4, due to the lack of knowledge of the capacity.

The difference in rational beliefs over social learning will affect the provider's profit. In the following, we first analyze the provider's optimal dynamic pricing policy, and then examine the impact of the participation-dependent social learning on the optimal pricing policy and profit.

*4.2.1 The Provider's Pricing Policy.* In the first period, the provider chooses $p_1^*$ to maximize its expected total profit calculated as follows:

$$\pi(p_1) = p_1 \cdot \max\left\{0, 1 - \frac{p_1}{q_0}\right\} + \int_0^\infty \frac{q_u}{4} f_{\text{NI}}(q_u; n_1) dq_u. \tag{20}$$

Here $f_{\text{NI}}(q_u; n_1)$ is the density function in Lemma 4.6. We can derive the equilibrium two-period pricing policy for the uninformed provider and congestion-unaware users similar as in Algorithm 1, with different preposterior belief in Lemma 4.6 and total profit in (20). We summarize the uninformed provider's equilibrium pricing policy in the following theorem.

Theorem 4.7. *In the presence of the participation-dependent social learning, there exits a unique equilibrium in the two-period stochastic game played between the uninformed provider and congestion-unaware users. More specifically,*

(a) *In the first period, the equilibrium price $p_1^*$ is independent of the social learning parameter $\gamma$.*
(b) *In the second period, the provider chooses $p_2^*$ to be*

$$p_2^*(q_u) = \begin{cases} \frac{q_u}{2}, & \text{if } q_u > 0; \\ 0, & \text{if } q_u \le 0. \end{cases}$$

Proof. See Appendix F in the technical report [26]. □

[13] The subscript "NI" stands for "not informed" or "uninformed".

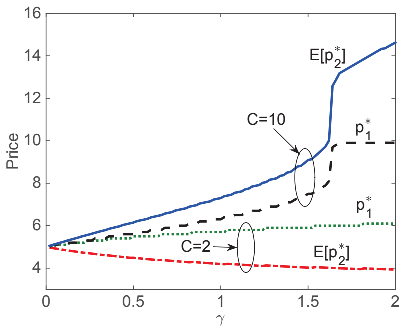


**Figure 3: Optimal prices for the informed provider and congestion-unaware users**

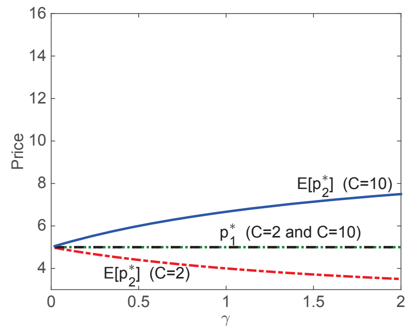


**Figure 4: Optimal prices for the uninformed provider and congestion-unaware users**

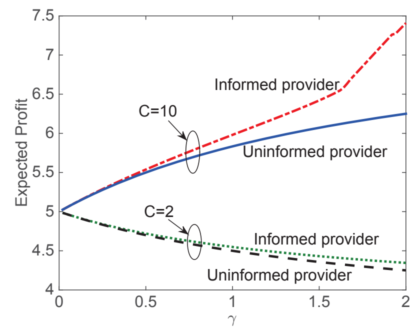


**Figure 5: Optimal expected profit with congestion-unaware users**

4.2.2 *Impact of Participation-Dependent Social Learning on the Optimal Pricing Policy and Profit.* We examine the impact of $\gamma$ on the uninformed provider's expected optimal pricing policy.

Fig. 4 shows the impact of $\gamma$ on the optimal pricing policy for the uninformed provider under different values of $C$. We set $q_0 = 10$ and $\sigma^2 = 1$. Compared with Fig. 3, we can see that $p_1^*$ and $E[p_2^*]$ for the uninformed provider are lower than the ones for the informed provider, respectively. When the provider does not have the capacity information, its pricing becomes more conservative.

Fig. 5 shows the impact of $\gamma$ and $C$ on the optimal expected profit, for both the informed provider and uninformed provider, under the optimal pricing policies shown in Fig. 3 and Fig. 4, respectively. We have two key observations. First, for both information scenarios, the optimal profit increases with $\gamma$ when $C$ is large ($C = 10$) and decreases with $\gamma$ when $C$ is small ($C = 2$). In other words, the participation-dependent social learning is beneficial to the provider when the capacity is large. The reason is that as $\gamma$ increases, the reviews can more accurately reveal the information regarding the large capacity, which attracts subscribers in the second period and improves the provider's profit. Second, the informed provider can gain more profit compared with the uninformed provider, which implies that the knowledge of the capacity information can help the provider make better pricing decisions.

## 5 DYNAMIC PRICING FOR CONGESTION-AWARE USERS

In this section, we analyze the two-period game for congestion-aware users, i.e., users are aware of the dependence of QoS on the congestion. More specifically, users know the function $\hat{q}(C, n_t) = C/n_t$, although they do not know the capacity $C$ at the beginning of the first period. Hence only second-period subscribers can take advantage of the congestion-awareness through the learning. Similar as Section 4, we will first analyze the informed provider scenario, and then analyze the uninformed provider scenario.

### 5.1 Informed Provider with Congestion-Aware Users

5.1.1 *Second-Period Subgame.* Congestion-aware users can learn the capacity information from the first-period reviews. More specifically, users have a belief of the capacity, $\bar{C}$, from the updated mean QoS $q_u$ (as described in Lemma 4.1) and the relationship $q_u = \bar{C}/n_1$. Hence,

$$\bar{C} = q_u \cdot n_1.$$

Since each user's second-period subscription decision depends on the congestion level, that is, other users' subscription decisions, users' second-period interactions form a subscription game, of which the *players* are all users. The *strategy* of each user $i$ is a binary variable $s_i \in \{0, 1\}$, where $s_i = 1$ means that user $i$ subscribes to the provider in the second period, and $s_i = 0$ means that he does not. Under a given strategy profile $\boldsymbol{s} = \{s_i : \forall i\}$, the fraction of users who subscribe in the second period $n_2$ is the size of the set $\{i : s_i = 1\}$, and the expected *payoff* of user $i$ is

$$E[v_{i2}] = a_i q_2 - p_2 = a_i \cdot \frac{\bar{C}}{n_2} - p_2 = a_i \cdot \frac{q_u n_1}{n_2} - p_2. \quad (21)$$

Given other users' strategies $\boldsymbol{s}_{-i} = \boldsymbol{s} \setminus \{s_i\}$, user $i$'s *best response* is to subscribe to the provider if $E[v_{i2}] \geq 0$, i.e.,

$$a_i \geq \frac{p_2}{q_2} = \frac{p_2 n_2}{q_u n_1}.$$

At equilibrium, the second-period subscription satisfies $n_2^* = 1 - \frac{p_2 n_2^*}{q_u n_1}$, which leads to

$$n_2^* = \frac{q_u n_1}{p_2 + q_u n_1}. \quad (22)$$

The provider's second-period profit is

$$\pi_2(p_2) = p_2 \cdot n_2^* = p_2 \cdot \frac{q_u n_1}{p_2 + q_u n_1} = \frac{q_u n_1}{1 + \frac{q_u n_1}{p_2}}, \quad (23)$$

which increases with $p_2$. Hence, the provider will choose its price in the second period equal to the maximum value possible. We assume that there is a fixed maximum price for the provider's second-period price, $\bar{p}_2$. Such a maximum price can arise due to market competition or government regulations [15].

We describe the unique equilibrium of the second-period subgame as follows.

Lemma 5.1. *Given any $q_u$, there exists a unique equilibrium in the second-period subgame played between the informed provider and congestion-aware users. More specifically:*

- *The provider's optimal second-period pricing policy is*

$$p_2^*(q_u) = \bar{p}_2. \quad (24)$$

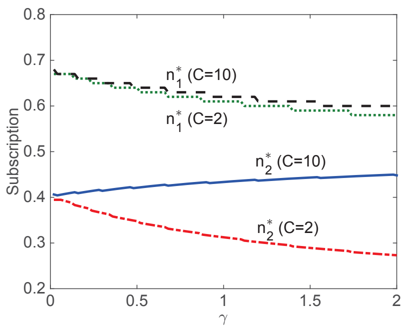


**Figure 6: Congestion-aware users' optimal subscriptions (informed provider)**

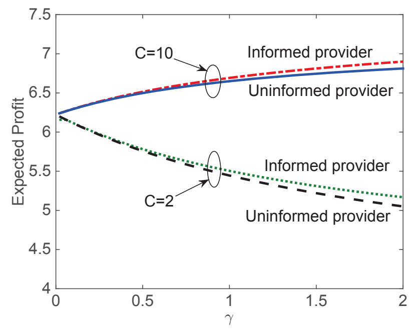


**Figure 7: Optimal expected profit (congestion-aware users)**

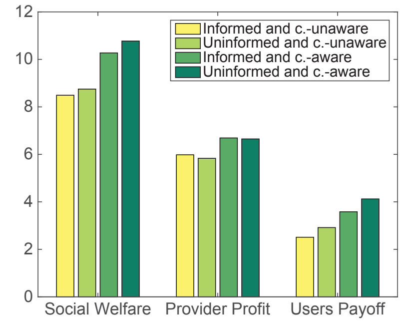


**Figure 8: Comparison of four information scenarios**

- *User $i$ subscribes to the provider if and only if*

$$a_i \geq \frac{p_2}{p_2 + q_u n_1}.$$

- *The provider's optimal second-period profit is*

$$\pi_2^*(p_2^*; q_u) = \frac{q_u n_1}{1 + q_u n_1/\bar{p}_2}. \tag{25}$$

Proof. See Appendix G in the technical report [26]. □

*5.1.2 First-Period Subgame.* The analysis of users' first-period subscription decisions is the same as the one in Section 4.1.

Next we consider the provider's pricing decision. In the first period, taking into account the users' subscription choice as a function of $p_1$, as well as the probabilistic equilibrium of the second-period subgame, the provider chooses $p_1^*$ to maximize its expected total profit in two periods:

$$\pi(p_1) = p_1 \cdot \max\left\{0, 1 - \frac{p_1}{q_0}\right\} + \int_0^\infty \frac{q_u n_1}{1 + q_u n_1/\bar{p}_2} f_I(q_u; n_1) dq_u. \tag{26}$$

Here $f_I(q_u; n_1)$ is the density function described in Lemma 4.4.

Theorem 5.2. *In the presence of the participation-dependent social learning, there exits a unique equilibrium in the two-period stochastic game played between the informed provider and congestion-aware users. More specifically,*

(a) *In the first period, $p_1^* < q_0$, i.e., it is never optimal for the provider to choose a first-period price that results in no subscription in the first period.*
(b) *In the second period, the provider chooses the maximum price $p_2^* = \bar{p}_2$.*

Proof. See Appendix H in the technical report [26]. □

*5.1.3 Impact of Participation-Dependent Social Learning on Users' Subscriptions.* We numerically explore the users' subscription decisions, which demonstrate the key difference between the congestion-aware users here and the congestion-unaware users in Section 4. Fig. 6 shows the congestion-aware users' subscription decisions.

First, Fig. 6 shows that the first-period subscription $n_1^*$ always decreases with $\gamma$ regardless of $C$. The reason is that as $\gamma$ increases, the posterior QoS belief depends more on user reviews. Recall that a larger $n_1^*$ will cause more severe congestion and a lower QoS. To induce a higher rating from user reviews and affect the congestion-aware users in the second period, the provider will increase its price $p_1^*$ with $\gamma$.

Second, Fig. 6 shows that when the capacity $C$ is small ($C = 2$), the second-period subscription fraction $n_2^*$ decreases with $\gamma$; however, when the capacity $C$ is large ($C = 10$), $n_2^*$ increases with $\gamma$. The reason is that as $\gamma$ becomes larger, the QoS information revealed through user reviews is more accurate. Hence, congestion-aware users have a better knowledge of the capacity $C$, and make better subscription decisions accordingly, i.e., fewer subscriptions for a small $C$ and more subscriptions for a large $C$. This is very different from the congestion-unaware users' case in Section 4, where the second-period subscription fraction $n_2^*(p_2^*) = 1/2$ is fixed.

## 5.2 Uninformed Provider with Congestion-Aware Users

Now we analyze the uninformed provider scenario. The analysis of the second period is the same as Section 5.1, as in Lemma 5.1. For the first period, the analysis of users' subscription is the same as Section 4.1, as in Lemma 4.3. The key difference from Section 5.1 is the provider's pricing decision in the first period.

*5.2.1 The Provider's Pricing Policy.* In the first period, the provider chooses $p_1^*$ to maximize its expected total profit:

$$\pi(p_1) = p_1 \cdot \max\left\{0, 1 - \frac{p_1}{q_0}\right\} + \int_0^\infty \frac{q_u n_1}{1 + q_u n_1/\bar{p}_2} f_{NI}(q_u; n_1) dq_u. \tag{27}$$

Here $f_{NI}(q_u; n_1)$ is the density function in Lemma 4.6.

Theorem 5.3. *In the presence of the participation-dependent social learning, there exits a unique equilibrium in the two-period stochastic game played between the uninformed provider and congestion-aware users. More specifically,*

(a) *In the first period, the equilibrium price $p_1^*$ is independent of the social learning parameter $\gamma$.*
(b) *In the second period, the provider chooses the maximum price $p_2^* = \bar{p}_2$.*

Proof. See Appendix I in the technical report [26]. □

*5.2.2 Impact of Participation-Dependent Social Learning on the Provider's Profit and Users' Payoffs.* Fig. 7 shows the impact of $\gamma$ and

$C$ on the optimal expected profit, for both the informed provider and the uninformed provider, when users are congestion-aware.[14] As we can see, the provider with a large capacity ($C = 10$) benefits from the learning. Moreover, whether the provider is informed or not has a relatively small impact on its profit.

In Fig. 8, we compare the four information scenarios regarding social welfare, the provider's profit, and users' total payoff.[15] We choose $q_0 = 10, \sigma_0^2 = 1, \sigma^2 = 1$, and $C = 10$. The comparison for the small capacity case ($C = 2$) is similar, and we present the figure in Appendix J in [26] due to the space constraint. Compared with the congestion-unaware users scenarios, the social welfare, the provider's profit, and users' payoff are all higher in congestion-aware users scenarios. This is because users can make better subscription decisions based on their knowledge of congestion, and the provider can extract more profit from users based on its anticipation of users' response to the pricing policy. Hence, both the provider and users benefit from users' awareness of the congestion effect, regardless of the capacity value $C$. This implies that the provider should make the users aware of the congestion effect whenever possible, which will benefit everyone in the market.

# 6 CONCLUSION

In this paper, we analyzed the Internet-based service provider's dynamic pricing problem with participation-dependent social learning. The learning object, the QoS, depends on not only the resource capacity but also the congestion effect. We studied four scenarios, depending on whether the provider has an accurate prior knowledge of the capacity and whether users are aware of the congestion effect. We showed that when the capacity is small or the prior QoS belief is high, the provider will set a first-period introductory price higher than the (expected) second-period regular price. Furthermore, participation-dependent social learning is beneficial to the provider with a large capacity. There are several interesting directions to explore in the future. For example, it would be interesting to study multiple providers' competition with participation-dependent social learning.

# REFERENCES


[1] M. H. Afrasiabi and R. Guérin. 2012. Pricing strategies for user-provided connectivity services. In *Proc. of IEEE INFOCOM*. 2766–2770.

[2] A. Beloglazov and R. Buyya. 2013. Managing overloaded hosts for dynamic consolidation of virtual machines in cloud data centers under quality of service constraints. *IEEE TPDS* 24, 7 (2013), 1366–1379.

[3] Blizzard. 2017. World of Warcraft: Subscriptions. Retrieved 2017 from https://us.shop.battle.net/en-us/product/world-of-warcraft-subscription

[4] S. Bose, G. Orosel, M. Ottaviani, and L. Vesterlund. 2008. Monopoly pricing in the binary herding model. *Economic Theory* 37, 2 (2008), 203–241.

[5] C. G. Brinton, S. Buccapatnam, M. Chiang F. M. F. Wong, and H. V. Poor. 2016. Social learning networks: Efficiency optimization for MOOC forums. In *Proc. of IEEE INFOCOM*.

[6] C. G. Brinton, M. Chiang, S. Jain, H. Lam, Z. Liu, and F. M. F. Wong. 2014. Learning about social learning in MOOCs: From statistical analysis to generative model. *IEEE Trans. on Learning Technologies* 7, 4 (2014), 346–359.

[7] K. T. Chen, P. Huang, G. S. Wang, C. Y. Huang, and C. L. Lei. 2006. On the sensitivity of online game playing time to network QoS. In *Proc. of IEEE INFOCOM*.

[8] X. Chen, X. Gong, L. Yang, and J. Zhang. 2017. Amazon in the white space: Social recommendation aided distributed spectrum access. *IEEE/ACM Transactions on Networking* 25, 1 (2017), 536–549.

[9] M. Chiang. 2012. *Networked Life*. Cambridge University Press.

[10] Alibaba Cloud. 2017. Alibaba Cloud forum. Retrieved 2017 from https://www.alibabacloud.com/forum?spm=a3c0i.7911826.675768.dnavsupport4.f7cc7a5zRiJ06

[11] Alibaba Cloud. 2017. SSD cloud server ECS starter package plan. Retrieved 2017 from https://www.alibabacloud.com/?spm=a3c0i.138096.675768.1.296c2ca8l9uwsz

[12] Coursera. 2016. Introducing subscriptions for specializations. Retrieved 2016 from https://blog.coursera.org/introducing-subscriptions-for-specializations/

[13] G. Darzanos, I. Koutsopoulos, and G. D. Stamoulis. 2016. Economics models and policies for cloud federations. In *Proc. of IFIP*. 485–493.

[14] M. H. DeGroot. 2005. *Optimal Statistical Decisions*. John Wiley and Sons.

[15] I. M. Dobbs. 2004. Intertemporal price cap regulation under uncertainty. *The Economic Journal* 114, 495 (2004), 421–440.

[16] L. Duan, J. Huang, and B. Shou. 2010. Cognitive mobile virtual network operator: Investment and pricing with supply uncertainty. In *Proc. of IEEE INFOCOM*.

[17] Google Project Fi. 2017. Plan features. Retrieved 2017 from https://fi.google.com/about/plan/

[18] L. Gao, G. Iosifidis, J. Huang, and L. Tassiulas. 2014. Hybrid data pricing for network-assisted user-provided connectivity. In *Proc. of IEEE INFOCOM*.

[19] A. Ghosh and J. Kleinberg. 2013. Incentivizing participation in online forums for education. In *Proc. of ACM Conference on Electronic Commerce*. 525–542.

[20] L. Huang, X. Liu, and X. Hao. 2014. The power of online learning in stochastic network optimization. In *Proc. of ACM SIGMETRICS*. 153–165.

[21] B. Ifrach, C. Maglaras, and M. Scarsini. 2014. Bayesian social learning with consumer reviews. In *Proc. of ACM SIGMETRICS*.

[22] A. Le, L. Keller, C. Fragouli, and A. Markopoulou. 2012. MicroPlay: a networking framework for local multiplayer games. In *Proc. of the first ACM international workshop on Mobile Gaming*. 13–18.

[23] T. N. Le, V. G. Subramanian, and R. A. Berry. 2016. Are imperfect reviews helpful in social learning?. In *Proc. of IEEE ISIT*. 2089–2093.

[24] K. Liu and Q. Zhao. 2012. Cooperative game in dynamic spectrum access with unknown model and imperfect sensing. *IEEE Transactions on Wireless Communications* 11, 4 (2012), 1596–1604.

[25] Y. Liu and M. Liu. 2015. An online learning approach to improving the quality of crowd-sourcing. In *Proc. of ACM SIGMETRICS*. 217–230.

[26] Q. Ma, B. Shou, J. Huang, and T. Başar. 2017. Online technical report. http://jianwei.ie.cuhk.edu.hk/publication/ApendixMobihoc18Learning.pdf

[27] R. T. Ma. 2016. Usage-based pricing and competition in congestible network service markets. *IEEE/ACM Transactions on Networking* 24, 5 (2016), 3084–3097.

[28] S. T. Maguluri, R. Srikant, and L. Ying. 2012. Stochastic models of load balancing and scheduling in cloud computing clusters. In *Proc. of IEEE INFOCOM*. 702–710.

[29] Y. Papanastasiou and N. Savva. 2016. Dynamic pricing in the presence of social learning and strategic consumers. *Management Science* 63, 4 (2016), 919–939.

[30] J. D. Pellegrino and C. Dovrolis. 2003. Bandwidth requirement and state consistency in three multiplayer game architectures. In *Proc. of ACM NetGames*. 52–59.

[31] V. R. Raja, V. Ramaswamy, S. Shakkottai, and V. Subramanian. 2016. Mean field equilibria of pricing games in Internet marketplaces. In *Proc. of ACM SIGMETRICS*. 387–388.

[32] J. Song and R. Guérin. 2017. Pricing and bidding strategies for cloud computing spot instances. In *Proc. of IEEE INFOCOM Smart Data Pricing (SDP) Workshop*.

[33] W. Wang, L. Ying, and J. Zhang. 2015. A game-theoretic approach to quality control for collecting privacy-preserving data. In *Proc. of IEEE Allerton*. 474–479.

[34] Wikipedia. 2017. Virtual private server. Retrieved 2017 from https://en.wikipedia.org/wiki/Virtual_private_server

[35] H. Xie, R. T. Ma, and J. Lui. 2018. Enhancing reputation via price discount in e-commerce systems: A data driven approach. *ACM Transactions on Knowledge Discovery from Data (TKDD)* 12, 3 (2018), 1–29.

[36] H. Xu, D. Liu, H. Wang, and A. Stavrou. 2015. E-commerce reputation manipulation: The emergence of reputation-escalation-as-a-service. In *Proc. of WWW*.

[37] X. Yao, R. Zhang, Y. Zhang, and Y. Lin. 2017. Verifiable social data outsourcing. In *Proc. of IEEE INFOCOM*.

[38] M. Yu, L. Debo, and R. Kapuscinski. 2015. Strategic waiting for consumer-generated quality information: Dynamic pricing of new experience goods. *Management Science* 62, 2 (2015), 410–435.

[39] T. Zhang, H. Wu, X. Liu, and L. Huang. 2016. Learning-aided scheduling for mobile virtual network operators with QoS constraints. In *Proc. of IEEE WiOpt*.

[40] X. Zhang, L. Cui, and Y. Wang. 2014. Commtrust: Computing multi-dimensional trust by mining e-commerce feedback comments. *IEEE TKDE* 26, 7 (2014), 1631–1643.

[41] L. Zheng, C. Joe-Wong, C. G. Brinton, C. W. Tan, S. Ha, and M. Chiang. 2016. On the viability of a cloud virtual service provider. In *Proc. of ACM SIGMETRICS*. 235–248.

[42] L. Zheng, C. Joe-Wong, J. Chen, C. G. Brinton, C. W. Tan, and M. Chiang. 2017. Economic viability of a virtual ISP. In *Proc. of IEEE INFOCOM*.


[14] Compared with the informed provider scenario, the main difference of users' subscription for the uninformed provider scenario is that $n_1^*$ does not change with $\gamma$, and hence we omit the figure for users' subscriptions.

[15] The social welfare is the summation of the provider's profit and users' total payoff.