

# Electrical Vehicle Charging Station Profit Maximization: Admission, Pricing, and Online Scheduling

Shuoyao Wang, Suzhi Bi, *Member, IEEE*, Ying-Jun Angela Zhang, *Senior Member, IEEE*, and Jianwei Huang, *Fellow, IEEE*

**Abstract**—The rapid emergence of electric vehicles (EVs) demands an advanced infrastructure of publicly accessible charging stations that provide efficient charging services. In this paper, we propose a new charging station operation mechanism, the JoAP, which jointly optimizes the EV admission control, pricing, and charging scheduling to maximize the charging station’s profit. More specifically, by introducing a tandem queueing network model, we analytically characterize the average charging station profit as a function of the admission control and pricing policies. Based on the analysis, we characterize the optimal JoAP algorithm. Through extensive simulations, we demonstrate that the proposed Joint Admission and Pricing algorithm (JoAP) on average can achieve 330% and 531% higher profit than a widely adopted benchmark method under two representative waiting-time penalty rates.

**Index terms**— Admission and schedule, electrical vehicle, smart grid, pricing, queueing analysis

## NOMENCLATURE

### Set

$\mathcal{V}$  The set of all EVs that arrive at the parking station during the time period of interest

$\Pi$  The feasible set of the admission control policies

### Notation

$\alpha$  The fixed charging power

$\beta$  The elasticity parameter of the utility function  $U(d)$

$\lambda_1, \lambda_2, \gamma$  The parameters of the mixture exponential distribution

$\omega_{\pi_n}(d)$  The average waiting time under  $\pi_n$  and  $d$

$\pi_n$  A proposed admission control policy involving  $n$  sub-processes

$\rho$  The load density admitted to the charging station

$\varphi$  The battery capacity of an individual EV

$c$  The average waiting time penalty rate

$d_i$  The charging demand of EV  $i$

$f_{\text{Ph}}(x)$  The PDF of the approximated inter-arrival time of admitted arrivals of Q.2

$F_X(x)$  The CDF of the inter-arrival time of admitted arrivals of Q.2

$f_X(x)$  The PDF of the inter-arrival time of admitted arrivals of Q.2

$h(\omega)$  The penalty when the average waiting time is  $\omega$

$m$  The number of charging ports

$P_i(n, d)$  The steady-state probability of state  $i$  under  $n$  and  $d$

$P_{\pi_n}(d)$  The average admission probability under  $\pi_n$  and  $d$

$T_v$  The minimum inter-arrival time of one sub-process

$U(d)$  The utility function of EVs

$X$  The inter-arrival time of admitted arrivals of Q.2

$x_i^{\mathcal{V}}(\pi_n, d)$  The binary admission decision of EV  $i$

$Y$  The coordinated inter-arrival time of admitted arrivals of the  $GI^{(m^*)}/D/1$  queue

### Variable

$n$  The number of sub-process in Q.1

$r$  The charging price announced by the charging station

## I. INTRODUCTION

**E**NVIRONMENTAL awareness and the rising fuel cost have stimulated an increasing interest in electrical vehicles (EVs). Establishing a conveniently available public charging infrastructure is essential to ensure a large market penetration of EVs [1]. Currently, however, many charging facilities are not yet profitable due to low expected revenues, high capital expenditures, and high operating and maintenance costs [2].

In light of this, recent studies have focused on improving the operation efficiency of EV charging stations (e.g., [3]-[7]) by carefully designing the charging scheduling and pricing mechanisms. In particular, You and Yang in [3] characterized an optimal offline charging scheduling scheme, where “offline” means that the scheduling decision relies on the noncausal information of future EV charging profiles. Tang and Zhang in [4] relaxed the assumption of noncausal information by utilizing only the statistical distributions, instead of the exact realizations, of future EV charging profiles. In [5], Tang *et al.* designed an online charging scheduling algorithm that does not require any future information, not even the distributional information. Refs. [6] and [7] further proposed charging scheduling and pricing schemes to incentivize EV users to maximize the social welfare, i.e., minimizing the network-wide charging cost or maximizing the total economic surplus.

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Most existing studies, e.g. [3]-[6], assumed that a charging station has unlimited charging power to accommodate an infinite number of EVs simultaneously. In practice, however, the total charging power is bounded due to physical and security constraints of the distribution network. Moreover, the number of EVs that a charging station can accommodate is limited by the hardware and space constraints. As such, a charging waiting time (defined as the time between the arrival time of the EV to the charging station and the time that the EV starts to receive service) is unavoidable, which negatively impacts the users' experience. Hence, it is necessary to implement an effective admission control policy to reduce the impact of excessive charging waiting time due to random EV arrivals.

A practically-adopted admission control is the queue-length based admission (QBA) policy, where a newly arrived EV is admitted as long as the number of EVs waiting to be served at the station is below a specific threshold. However, such a policy performs poorly in many cases, as illustrated in Section V. In contrast, Wei *et al.* in [8] proposed an admission control scheme, where the admission decision is based on the charging demands of EVs that have already arrived. The unknown future charging demands, however, were not considered in [8], resulting in poor profit performance in practical scenarios (see Section V for related examples).

In this paper, we propose a novel EV charging station operating mechanism that jointly optimizes pricing, charging scheduling, and admission control. The proposed algorithm, referred to as JoAP (joint admission control and pricing), maximizes the average profit of a charging station. Here, the profit is defined as the difference between the revenue and a penalty proportional to the average charging waiting time. The waiting time penalty reflects the EV owners' impatience of waiting in the queue for an excessively long time, which undermines the reputation of the charging station and reduces its long-term profit. In the JoAP algorithm, each EV user maximizes its surplus by adjusting its charging demand in response to the charging price. Meanwhile, the charging station maximizes its profit by choosing the proper admission control, scheduling and pricing policies. The contributions of this paper are summarized as follows:

- 1) *Admission control, scheduling, and pricing scheme:* To the best of our knowledge, this is the first paper that jointly optimizes pricing, scheduling, and admission control of an EV charging station. In particular, we propose a novel multi-sub-process based admission control scheme, which allows us to flexibly tradeoff between the revenue of the charging station and the waiting time of the EVs.
- 2) *Tandem queueing model:* We propose a tandem queueing model to analytically characterize the performance of JoAP. More specifically, we obtain closed-form expressions of the average waiting time and the admission probability as functions of the chosen algorithm parameters.
- 3) *Optimization of algorithm parameters:* Based on the tandem queue analysis, we propose a low-complexity algorithm to compute the close-to-optimal parameters of the JoAP algorithm. Our simulations show that JoAP algorithm on average achieves 330% and 531% higher

profit than a widely adopted benchmark method under two representative waiting-time penalty rates.

The rest of this paper is organized as follows. In Section II, we introduce the system model and formulate the problem. In Section III, we analyze the impact of the admission control policy on the admission probability and the average waiting time. In Section IV, we propose an algorithm to simultaneously maximize the charging station's profit and individual EV user's payoff surplus. Simulation results are presented in Section V. Finally, we conclude this paper in Section VI.

## II. SYSTEM MODEL

### A. Charging Station Operation

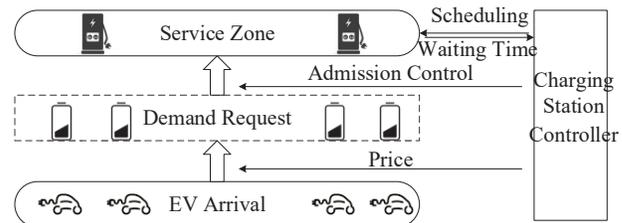


Fig. 1: The proposed charging station interaction system

We consider a charging station with  $m$  charging ports and a sufficiently large number of parking lots, i.e., much larger than  $m$  (Fig. 1). In this case, although a large number of EVs can be admitted to the charging station, at most  $m$  of them can be charged simultaneously because of the physical constraints of the power distribution network and safety concerns. The charging ports are connected to the parking lots through a switch scheduler, which allows real-time communications and controls between a particular charging port and a scheduled EV. For the simplicity of analysis, we assume that the cost of connecting EVs with charging ports is negligible. All charging ports operate with the same fixed charging power  $\alpha$ .

The charging station announces a charging price of  $r$  per unit energy to all arriving EVs. An EV  $i$ 's payment to the charging station is the product of  $r$  and the EV's demand  $d_i$ . A long waiting time negatively affects the EV users' experience, which may lead to customer churn in the long run. Thus, the charging station aims to determine the optimal pricing and admission control policy to maximize its average profit, which is the revenue minus the penalty due to EVs' waiting.

EVs arrive at the charging station according to a Poisson random process [8][9], and each EV expects the charging station to fulfill its demand as soon as possible. When an EV  $i$  arrives, it attempts to maximize its surplus by choosing its charging demand  $d_i$  according to the charging price  $r$ . Based on the requested demand  $d_i$ , the charging station decides whether to admit the EV. The charging station optimizes the admission control policy to avoid excessive delay of admitted EVs. Once admitted, the EVs are charged on a first come first serve (FIFO) basis. It has been shown in [10] that, when all EVs have the same demand, the FIFO policy is equivalent to the shortest job first policy, and therefore is optimal in terms of minimizing the average waiting time.

### B. Optimization from EVs' Perspective

Suppose that each EV has a battery capacity  $\varphi$  and a utility function  $U(d)$  that measures its satisfaction when a charging demand  $d$  is fulfilled. In general,  $U(d)$  is an increasing concave function. Here, for simplicity, we adopt the following increasing concave function [11][12], where  $\beta$  is the elasticity parameter and  $U(\varphi)$  is the maximum utility an EV can receive.

$$U(d) = U(\varphi) \frac{1 - e^{-\beta d}}{1 - e^{-\beta \varphi}}, \quad \forall 0 \leq d \leq \varphi. \quad (1)$$

In particular,  $U(d)$  represents a wide range of the user satisfaction. On one hand, EV users care about the charging demand much more than the charging price if  $\beta$  is close to zero. On the other hand, EV users are more price sensitive if  $\beta$  is large enough. Upon arrival, EV  $i$  decides its charging demand to maximize its customer surplus, i.e., utility minus payment, by solving

$$\max_{d_i} U(d_i) - r d_i \quad (2a)$$

$$\text{s.t. } 0 \leq d_i \leq \varphi. \quad (2b)$$

As Problem (2) is a concave maximization problem, we can compute the optimal demand  $d^*$  as a function of the charging price  $r$  as follows.

$$d^*(r) = \begin{cases} -\frac{\ln\left(\frac{1 - e^{-\beta \varphi}}{U(\varphi)\beta} r\right)}{\beta}, & \text{if } r \leq \frac{U(\varphi)\beta}{1 - e^{-\beta \varphi}}, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

We can show that  $d^*(r)$  is a decreasing function of the charging price  $r$  and becomes 0 when  $r$  is too high. This is consistent with the law of diminishing marginal utility (Gossen's First Law) in economics [13]. In this paper, we assume that the charging station knows the utility function [14]. Accordingly, the station can predict EV's demand  $d^*(r)$  in response to  $r$  as in (3). Thus, optimizing  $r$  is equivalent to optimizing  $d$  in the rest of the paper.

### C. Optimization from Charging Station's Perspective

Let  $\mathcal{V}$  denote the set of all EVs that arrive at the parking station during the time period of interest (e.g., 4 hours in our simulations). For each EV  $i \in \mathcal{V}$ , the charging station makes a binary admission decision  $x_i^{\mathcal{V}}(\pi_n, d)$ , where  $x_i^{\mathcal{V}}(\pi_n, d) = 1$  if EV  $i$  is admitted, and  $x_i^{\mathcal{V}}(\pi_n, d) = 0$  otherwise. Here,  $\pi_n$  denotes an admission policy, which will be detailed in Section III.A. Consequently, the average admission probability is

$$P_{\pi_n}(d) = E_{\mathcal{V}} \left[ \frac{1}{|\mathcal{V}|} \sum_{i \in \mathcal{V}} x_i^{\mathcal{V}}(\pi_n, d) \right], \quad (4)$$

where  $|\mathcal{V}|$  denotes the cardinality of  $\mathcal{V}$ . Moreover, the average waiting time achieved under policy  $\pi_n$  is a function of the demand  $d$  and the EV arrival process  $\mathcal{V}$ , denoted as  $\omega_{\pi_n}(\mathcal{V}, d)$ . Accordingly, the waiting time averaged over all the possible EV arrivals is denoted by

$$\omega_{\pi_n}(d) \triangleq E_{\mathcal{V}}[\omega_{\pi_n}(\mathcal{V}, d)]. \quad (5)$$

By satisfying an EV's charging demand  $d$ , the charging station receives a payment of  $rd$ , and pays an electricity cost

of  $p_e d$  to the utility company, where  $p_e$  is the electricity price. The penalty related to the average waiting time is denoted by  $h(\omega_{\pi_n}(d))$ , where  $h(\omega)$  is a general non-decreasing convex function of  $\omega$  [15]. Based on this, we formulate the charging station's profit-maximization problem as follows.<sup>1</sup>

$$\max_{\pi_n, d} P_{\pi_n}(d) (r - p_e) d - h(\omega_{\pi_n}(d)) \quad (6a)$$

$$\text{s.t. } d \geq 0, \quad i \in \mathcal{V}, \quad (6b)$$

$$\pi_n \in \Pi, \quad (6c)$$

$$d = -\frac{1}{\beta} \ln \left( \frac{1 - e^{-\beta \varphi}}{U(\varphi)\beta} r \right), \quad (6d)$$

where the feasible set  $\Pi$  will be introduced in Section III.A. The detailed expressions of  $P_{\pi_n}(d)$  and  $\omega_{\pi_n}(d)$  will be given in Section III.B and Section III.C, respectively.

## III. MULTI-SUB-PROCESS ADMISSION AND QUEUEING ANALYSIS

In this section, we first propose a multi-sub-process admission control scheme. Then, we present a tandem queueing model to analyze the impact of admission control policy and pricing decision on  $P_{\pi_n}(d)$  and  $\omega_{\pi_n}(d)$ .

### A. Admission Control and Queueing Model

The objective of admission control is to admit a large number of users with a guaranteed quality of service (QoS). Let us first consider an extreme case of complete arrival-process regulation, i.e., the inter-arrival time of two successively admitted EVs is always larger than a predefined threshold as the result of the admission control. If such a threshold is large enough, then the waiting time of every admitted EV is zero [10]. However, under this overly conservative policy, the station utilization can be very low, hence not achieving the maximum profit. To achieve a good balance among the waiting time, admission probability, and server utilization, we propose a multi-sub-process admission control scheme consisting of  $n$  sub-processes. In particular, the inter-arrival time of two consecutively admitted EVs of the same sub-process must be larger than a threshold, denoted as  $T_v$ . An EV is admitted as long as it can fit in one of the sub-processes. With some abuse of notations, we use  $\pi_n$  to denote the proposed admission control policy involving  $n$  sub-processes. Hence, the feasible set of all admission control policies considered in this paper is  $\Pi = \{\pi_n | n \in \mathcal{N}^+\}$ .

**Example 1.** Consider a  $\pi_2$ -admission policy that consists of two sub-processes, both having the same minimum inter-arrival time  $T_v$ , as shown in Fig. 2. When EV 1 arrives, we assign it to sub-process 1. When EV 2 arrives, we cannot assign to sub-process 1 as the inter-arrival time between EV 1 and EV 2 is shorter than  $T_v$  (the length of shadowed rectangle). Hence, we assign EV 2 to sub-process 2. For EV 3, we can assign it to sub-process 1. However, when EV 4

<sup>1</sup>Problem (6) does not consider the penalty of denying the EVs. However, we can consider this by simply adding a linear term of  $P_{\pi_n}(d)$  to the objective function. Doing so does not affect the structure of the problem, and our analysis will remain unchanged. For simplicity of exposition, this linear term is omitted for the time being.

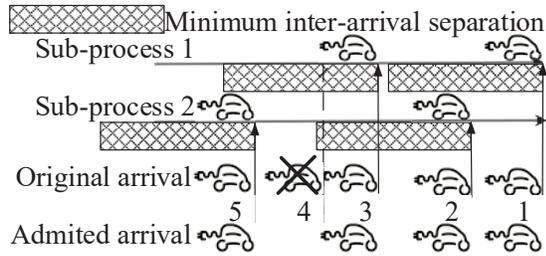


Fig. 2: Admission control example illustrated in Example 1

arrives, both sub-processes are “occupied”. Therefore, EV 4 has to be rejected. When EV 5 arrives, sub-process 2 becomes available again (due to the large enough inter-arrival time between EV 2 and EV 5). Hence, we accept EV 5 and assign it to sub-process 2.

We would like to emphasize that the multi-sub-process scheme can represent a wide range of admission control policies. On one hand, the admitted traffic is completely regulated if there is only one sub-process. On the other hand, when  $n$  approaches infinity, all EVs will be admitted regardless of the underlying distribution of the arrival process. Thus, choosing proper values of  $n$  and  $T_v$  allows us to balance the trade-off between the waiting time and admission probability, and eventually maximizes the charging station profit.

The admission process governed by the multi-sub-process scheme can be analyzed as a virtual queueing system with zero buffer and  $n$  servers, as represented by Q.1 (an  $M/T_v/n/n$  queue<sup>2</sup>) in Fig. 3. Each virtual server corresponds to a sub-process, which has a deterministic service time  $T_v$ . The arrival of Q.1 is the EV arrival process  $\mathcal{V}$ . As the buffer is zero for Q.1, an EV will be declined for service if it finds all virtual servers are busy (i.e., all sub-processes are occupied) upon arrival. Otherwise, the EV is admitted and will occupy an idle virtual server for a fixed time period of  $T_v$ . The departure from Q.1 means that the EV is admitted to the charging station.

Once the EVs are admitted, they are served in the charging station according to the FIFO policy. We model the queueing system in the charging station as Q.2 in Fig. 3, where the  $m$  charging ports represent  $m$  servers, each with a deterministic service time  $d/\alpha$ , where  $d$  and  $\alpha$  are the charging demand per EV and the fixed common charging rate per charging port, respectively. Note that the departure process of Q.1 is the arrival process of Q.2. To ensure the stability of Q.2, the inter-departure time of Q.1 must be greater than the average service time of Q.2, i.e.,  $nT_v > md/\alpha$ . We can equivalently represent this constraint as  $nT_v = \tau md/\alpha$ , where  $\tau > 1$ . To summarize, the determination of an admission control policy involves two decision variables,  $\tau$  and  $n$ , with which the charging station can compute  $T_v = \frac{\tau md}{n\alpha}$ . In the following, we

<sup>2</sup>We can represent a single queue using Kendall’s notation in the form  $A/S/C/K$ , where  $A$  describes the inter-arrival times,  $S$  describes the service time,  $C$  describes the number of servers, and  $C+K$  describes the number of spaces in the system. When the  $K$  parameter is not specified (e.g.  $M/M/1$  queue), it is assumed that  $K = \infty$ . In Kendall’s notation:  $M$  stands for Markov or memoryless process,  $D$  stands for deterministic process,  $G$  stands for general and corresponds to an arbitrary probability distribution,  $Ph$  stands for phase-type process, and  $\cdot$  stands for any process.

will consider optimizing  $n$  under a fixed value of  $\tau$ . Without loss of generality, we assume that  $\alpha$  equals 1. We will examine the impact of  $\tau$  in Section V.

In practice, a well regulated arrival process seldom yields a long queue length [16]. Consequently, we ignore the impact of buffer of Q.2 and assume that it is infinite in the following analysis. In the remaining of this section, we are going to analyze the performance of the  $\underbrace{M/T_v/n/n}_{Q.1} + \underbrace{\cdot/d/m}_{Q.2}$  tandem

queueing network. The analysis will be useful in designing the JoAP algorithm in Section IV.

Before concluding this subsection, we would like to emphasize that Q.1 in Fig. 3 is a virtual queue that does not exist in reality. We consider Q.1 for the purpose of the admission control. Queue Q.2 is a real queue corresponding to the service in the charging station. As such, the admission probability is the probability that a new arrival is admitted to Q.1, and the charging waiting time is the waiting time in Q.2.

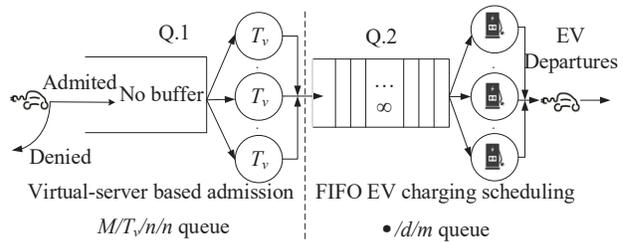


Fig. 3: The tandem queueing network model

### B. Admission Probability

Previous queueing literature (e.g., [17]) has numerically analyzed the performance of  $M/D/n/n+K$  queues (e.g., Q.1 in Fig. 3) without analytical characterization of the system performance. Tijms in [18] showed that a two-phase process server can be used to approximate a deterministic server with a marginal performance gap. Based on this approximate model, we derive a closed-form expression of steady-state probabilities of Q.1 in the following Lemma 1. To the best of our acknowledgment, this paper is the first analytical study of the  $M/D/n/n+K$  system with  $K = 0$  (i.e., zero buffer).

**Lemma 1.** Consider an  $M/D/n/n$  queue with a Poisson process with a arrival rate  $\lambda$ , a deterministic service time  $D = \tau md/n$ , and zero buffer-size. The steady-state probability of state  $j$  (i.e., the probability that the system has  $j$  users being served simultaneously) can be calculated based on the two-phase-process approximation in [18] as follows,

$$P_j(n, d) = \frac{\left(\frac{d\tau m\lambda}{n}\right)^j}{j! \sum_{j'=0}^n \frac{\left(\frac{d\tau m\lambda}{n}\right)^{j'}}{j'!}}. \quad (7)$$

At any time, if a new EV arrives at the charging station, the probability that the charging station accepts the new arrival is equivalent to the probability that Q.1 is not full. Hence, the admission probability of Q.1 is:

$$P_{\pi_n}(d) = 1 - P_n(n, d) = 1 - \frac{\left(\frac{d\tau m\lambda}{n}\right)^n e^{-\frac{\tau md\lambda}{n}}}{\Gamma\left(n+1, \frac{\tau md\lambda}{n}\right)}. \quad (8)$$

We can prove Lemma 1 by induction and omit the detailed proof due to the page limit. The validity of Lemma 1 is verified in Fig. 4, where we compare the equation (8) with the

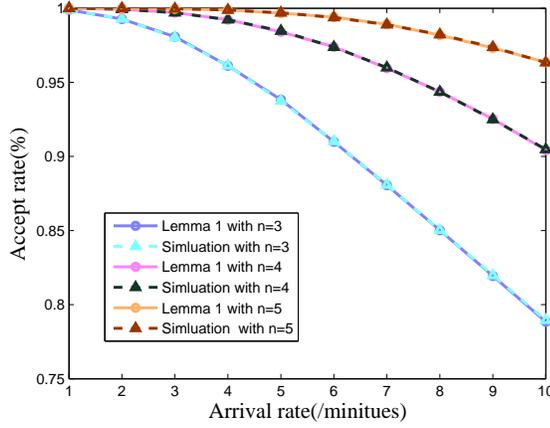


Fig. 4: The comparison of the admission probability between the simulation and Lemma 1, with  $\tau = 1.01$ ,  $m = 4$ ,  $\beta = 0.05$ ,  $\alpha = 3.3kW$ ,  $d = \varphi$ , and  $\gamma = 35kWh$

simulation results (without any approximation). We choose the number of servers in Q.1,  $n$ , to be 3, 4, and 5, respectively. Each point corresponds to the average over 1000 time periods. The maximum gap between the analysis and simulation is 0.01%, which verifies the accuracy of the results in Lemma 1.

### C. Average Waiting Time

1) *Admitted-arrival*: To study the average waiting time in Q.2, we derive the PDF (probability density function) of the inter-arrival time of Q.2.

**Lemma 2.** *The PDF of the inter-arrival time of admitted arrivals of Q.2 is*

$$f_X(x) = \begin{cases} \sum_{i=0}^n \frac{i}{T_v} \left( \frac{T_v - x}{T_v} \right)^{i-1} P_i(n, d), & \text{if } x \leq T_v, \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

*Proof.* Recall that the arrival process of Q.2 is the departure process of Q.1. According to [19], the residual service time of a queueing system is the service time remaining to a job under service when the system is observed at any time. The residual service time of Q.1 follows a uniform distribution in  $[0, T_v]$ , as the arrival process is memory-less (Poisson) and the buffer size is zero. When Q.1 is at a particular state  $i$ , the probability of no departure during the next period of time of a length  $x$  is equal to the probability that the residual service times of all existing jobs are no-less than  $x$ , i.e.,  $((T_v - x)/T_v)^i$ . Consequently, the probability of the first departure time (after the observation time point) being no greater than  $x$  is  $1 - ((T_v - x)/T_v)^i$ . Therefore, the CDF of the inter-departure time of Q.1 (i.e., the inter-arrival time of Q.2), denoted by  $X$ , is,

$$F_X(x) = \begin{cases} \sum_{i=0}^n \left( 1 - \left( \frac{T_v - x}{T_v} \right)^i \right) P_i(n, d), & \text{if } x \leq T_v, \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

Taking the derivative of (10) yields the PDF in Lemma 2.  $\square$

2) *Phase-type Approximation*: We now derive the average waiting time of Q.2 with the phase-type approximation. So far, there does not exist a general closed-form expression for the waiting time distribution of a  $GI/D/m$  queue, where  $GI$  means a general arrival process. To overcome this difficulty, [18] showed that the waiting time distribution of a  $GI/D/m$  queue is the same as that of a  $GI^{(m^*)}/D/1$  queue, where  $GI^{(m^*)}$  denotes a coordinated inter-arrival time process that is distributed as the sum of  $m$  inter-arrival times of a  $GI/D/m$  queue. Let  $Y$  denote the coordinated inter-arrival time of the  $GI^{(m^*)}/D/1$ . The mean and variance of  $X$  and  $Y$  are related by  $\mu_Y = m\mu_X$  and  $\sigma_Y^2 = m\mu_X^2 - m\mu_X^2$ .

Furthermore, a  $GI^{(m^*)}/D/1$  queue can be approximated by a  $Ph/D/1$  queue, where  $Ph$  means the phase-type process [18]. One of the most widely used phase-type distribution is the mixture exponential distribution, which is defined as the mixture of two exponential distributions with means  $1/\lambda_1$  and  $1/\lambda_2$ , and weights  $\gamma$  and  $1 - \gamma$ , respectively. Specifically, the PDF is given by

$$f_{Ph}(x) = \gamma e^{-\lambda_1 x} + (1 - \gamma) e^{-\lambda_2 x}. \quad (11)$$

In this paper, we replace the inter-arrival distribution of Q.2 with the mixture exponential distribution in (11). To ensure that the first and second moments of the mixture exponential distribution are equal to those of  $Y$ , we set  $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = 2\mu_Y$ ,  $\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} = \sigma_Y^2$ , and  $\gamma = \frac{1}{2}$ . In this way, we can approximate the waiting time distribution of Q.2 by that of the  $Ph/D/1$  queue. Furthermore, we derive in the following Theorem 1 the approximated average waiting time of the charging station. In the theorem, we will use notation  $\rho = \lambda P_{\pi_n}(d)d/m$  to denote the load density admitted to the charging station.

**Theorem 1.** *The approximated average waiting time at the charging station for the admitted EVs is*

$$\omega_{\pi_n}(d) = \frac{\rho d}{2(1-\rho)} [d^2 + 2d\mu_Y + \sigma_Y^2]. \quad (12)$$

Moreover,  $\omega_{\pi_n}(d)$  is an increasing convex function in  $d$ .

*Proof.* We can derive the approximated average waiting time based on Proposition 1 quoted in [18].

**Proposition 1.** [18] *For a  $Ph/D/1$  queue, let  $S$  and  $A$  denote the service time and the inter-arrival time, respectively. The Laplace transform  $a^*(s) = \int_0^\infty e^{-st} a(t) dt$  of the inter-arrival time  $A$  can be written as  $a^*(s) = \frac{a_1(s)}{a_2(s)}$ , where  $a(t)$  denotes the probability density function of  $S$  and  $a_1(s)$  and  $a_2(s)$  are two polynomials. Then, the average waiting time can be approximated as  $\frac{\rho E(S)}{2(1-\rho)} \left[ E(S^2) + E(A^2) + 2E(S) \frac{a_1'(0)}{a_1(0)} - 2\psi \frac{a_2'(0)}{a_2(0)} \right]$ , where  $\psi = \frac{a_2'(0) - a_1'(0)}{a_2(0)}$  and  $\rho$  is the load density.*

We apply Proposition 1 to our tandem queue model. As the Lapalaze transform of  $Y$  is  $\mathcal{L}\{f_Y(x)\} = \frac{\lambda_1 \lambda_2 + \frac{1}{2}(\lambda_1 + \lambda_2)}{(s + \lambda_1)(s + \lambda_2)}$ , we have  $a_1(s) = \lambda_1 \lambda_2 + \frac{1}{2}(\lambda_1 + \lambda_2)$ ,  $a_2(s) = (s + \lambda_1)(s + \lambda_2)$ ,  $\psi = \frac{\frac{1}{2}(\lambda_1 + \lambda_2)}{\lambda_1 \lambda_2}$ . Taking the first order derivative of  $\frac{\rho}{1-\rho}$  over  $d$ , we have  $\frac{\rho' d + \rho + \frac{\rho^2}{m}}{(1-\rho)^2}$ , which is a positive

increasing function in  $d$ . Substitute  $\lambda_1, \lambda_2$  with the representation of  $\mu_Y$  and  $\sigma_Y$ , we can express the average waiting time as  $d \left[ d^2 + EY^2 + 2d \frac{\frac{1}{2}\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} - 2 \frac{\frac{1}{2}\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \right] = d \left[ d^2 + EY^2 + 2d\mu_Y - \mu_Y^2 \right] = d \left[ d^2 + \sigma_Y^2 + 2d\mu_Y \right]$ . Notice that  $\mu_Y = \frac{m}{P_{\pi_n}(d)\lambda}$ . Consequently,  $d \left[ d^2 + \sigma_Y^2 + 2d\mu_Y \right] = d^3 + 2d\sigma_Y^2 + \frac{md^2}{P_{\pi_n}(d)}$  is a convex increasing function in  $d$  for fixed  $n$ . Thus,  $\omega_{\pi}(d)$  is a convex function in  $d$ , which is in agreement with Kingman's formula. Kingman's formula is a well-known approximation for the mean waiting time in a  $G/G/1$  queue, which states that  $w_{\pi}(d) \approx \frac{\rho d}{2(1-\rho)} [d^2 + EY^2]$ .  $\square$

Let us verify the approximation by comparing the average waiting time in (12) with simulation results (without any approximation). In Fig. 5, for each pair of arrival rate and individual demand, we simulate 1000 independent 1000-hour arrival processes  $\mathcal{V}$  and plot the average admission probabilities. The difference is no more than 0.1%.

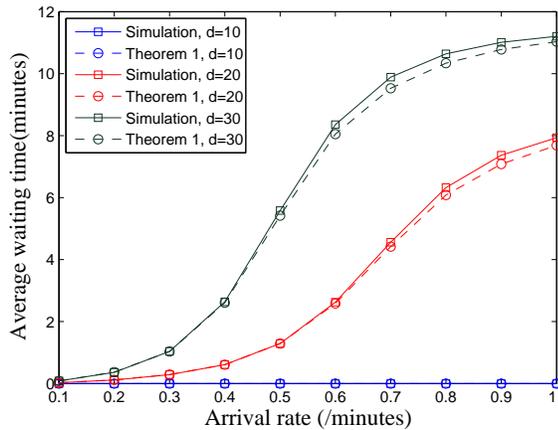


Fig. 5: The comparison of the average waiting time between the simulation and Theorem 1, with  $m = 4$ .

#### IV. OPTIMIZATION PROBLEM RECASTING AND PROFIT MAXIMIZATION

With the tandem queueing analysis, we rewrite (6) as

$$\max_{n,d} s(n, d) = P_{\pi_n}(d) \left( \frac{de^{-\beta d}}{\xi} - dp_e \right) - h(\omega_{\pi_n}(d)) \quad (13a)$$

$$\text{s.t. } 0 \leq d \leq \varphi, \quad n \in \mathcal{N}^+, \quad (13b)$$

where  $\xi = \frac{1-e^{-\beta\varphi}}{U(\varphi)\beta}$ ,  $P_{\pi_n}(d)$  and  $\omega_{\pi_n}(d)$  are given in (8) and (12), respectively. Recall that  $n$  is the number of sub-processes in Q.1 and  $d$  is the estimated EV demand in response to the charging price  $r$ . Variables  $n$  and  $d$  together determine the admission control and pricing strategy of the charging station. To solve the integer programming Problem (13) efficiently, we replace the decision variable  $n$  with  $P \triangleq P_{\pi_n}(d)$ . This is because for a particular feasible  $(P, d)$ , we can find a unique  $n$  that satisfies equation (8). Accordingly, (13) can be equivalently expressed as

$$\max_{P,d} \hat{s}(P, d) = P \left( \frac{de^{-\beta d}}{\xi} - dp_e \right) - h(\omega_{\pi_n}(d)) \quad (14a)$$

$$\text{s.t. } 0 \leq d \leq \varphi, \quad P \in (0, 1), \quad (14b)$$

$$P \in \{P_{\pi_n}(d) | \forall n \in \mathcal{N}^+, \forall d \in [0, \varphi]\}. \quad (14c)$$

Moreover, the objective function (14a) is concave under the conditions in (14b). Ref. [20] showed that the average waiting time of a  $GI/GI/1$  queue with first-come-first-served order is jointly convex in the effective-arrival-rate and the service rate. The effective-arrival-rate of the corresponding coordinated queue (a  $GI^{(m^*)}/D/1$  queue) of Q.2 is  $\frac{P\lambda}{m}$ . By the composition rule, we can see that  $-h(\omega_{\pi_n}(d))$  is jointly concave in  $(\frac{P\lambda}{m}, d)$  (thus in  $(P, d)$ ). This, together with the fact that  $P \left( \frac{de^{-\beta d}}{\xi} - dp_e \right)$  is jointly concave in  $(P, d)$ , implies that (14a) is a jointly concave function in  $(P, d)$ . If we ignore the integer constraint, then Problem (14) can be solved efficiently by the gradient method, with the optimal solution denoted as  $(P^v, d^v)$ . Accordingly,  $n^v$  can be obtained by solving (8) given  $(P^v, d^v)$ . However,  $n^v$  obtained through this approach does not necessarily satisfy the integer constraint. In the following Lemma 3, we show that the optimal solution to Problem (13) can be easily obtained by rounding  $n^v$  to the nearest integer. In the lemma, we will use the notation of  $d_n^* = \arg \max_d s(n, d)$ . Then, we have the following characterization of the optimal solution  $(n^*, d^*)$  to Problem (13).

**Lemma 3.** *Given that  $(n^v, d^v)$  is an optimal solution to Problem (14a-b), then the optimal solution to Problem (13) is either  $(\lfloor n^v \rfloor, d_{\lfloor n^v \rfloor}^*)$  or  $(\lceil n^v \rceil, d_{\lceil n^v \rceil}^*)$ , whichever yields the larger objective function value.<sup>3</sup>*

*Proof.* First, we show that for any  $\hat{n} < \lfloor n^v \rfloor$ ,  $s(\hat{n}, d_{\hat{n}}^*) \leq s(\lfloor n^v \rfloor, d_{\lfloor n^v \rfloor}^*)$ . It's equivalent to showing that for any  $\hat{n} < \lfloor n^v \rfloor$ , we can find an  $(\lfloor n^v \rfloor, d_1)$  such that  $s(\lfloor n^v \rfloor, d_1) \geq s(\hat{n}, d_{\hat{n}}^*)$ . From (6),  $P_{\pi_n}(d)$  is monotonically increasing in both  $n$  and  $d$ . Thus, we can always find a point  $(P_{\pi_{\lfloor n^v \rfloor}}(d_1), d_1)$  in the line segment between  $(P_{\pi_{\hat{n}}}(d_{\hat{n}}^*), d_{\hat{n}}^*)$  and  $(P^v, d^v)$ . The monotonicity of  $P_{\pi_n}(d)$  guarantees the existence and uniqueness of  $(P_{\pi_{\lfloor n^v \rfloor}}(d_1), d_1)$ . Due to the joint concavity of  $\hat{s}$  in  $(P, d)$ , we have  $\hat{s}(P^v, d^v) \geq \hat{s}(P_{\pi_{\lfloor n^v \rfloor}}(d_1), d_1) \geq \hat{s}(P_{\pi_{\hat{n}}}(d_{\hat{n}}^*), d_{\hat{n}}^*)$ . Due to the equivalence between Problem (11) and Problem (14), we have  $s(n^v, d^v) \geq s(\lfloor n^v \rfloor, d_1) \geq s(\hat{n}, d_{\hat{n}}^*)$ . Likewise, we can prove that for any  $\hat{n} > \lceil n^v \rceil$ ,  $s(\hat{n}, d_{\hat{n}}^*) \leq s(\lceil n^v \rceil, d_{\lceil n^v \rceil}^*)$ . Therefore, we can conclude that the optimal solution to Problem (13) is either  $s(\lfloor n^v \rfloor, d_{\lfloor n^v \rfloor}^*)$  or  $s(\lceil n^v \rceil, d_{\lceil n^v \rceil}^*)$ .  $\square$

Lemma 3 indicates that we can obtain the optimal  $n^*$  by rounding  $n^v$ . What remains is how to calculate  $d_{\lfloor n^v \rfloor}^*$  and  $d_{\lceil n^v \rceil}^*$  efficiently. The following Lemma 4 indicates that  $d_{\lfloor n^v \rfloor}^*$  and  $d_{\lceil n^v \rceil}^*$  can be easily obtained using single-variable convex optimization methods, e.g., the gradient search method.

**Lemma 4.** *Given  $n$ ,  $s(n, d)$  is concave in  $d$  for  $d \in \{d | \frac{1}{\xi} (e^{-\beta d} - d\beta e^{-\beta d}) - p_e \geq 0\}$ . Moreover,  $s(n, d)$  is concave in  $d$  when  $n = n^*$ .*

<sup>3</sup> $\lfloor n \rfloor$  and  $\lceil n \rceil$  denote the largest integer no greater than  $n$  and the smallest integer no less than  $n$ .

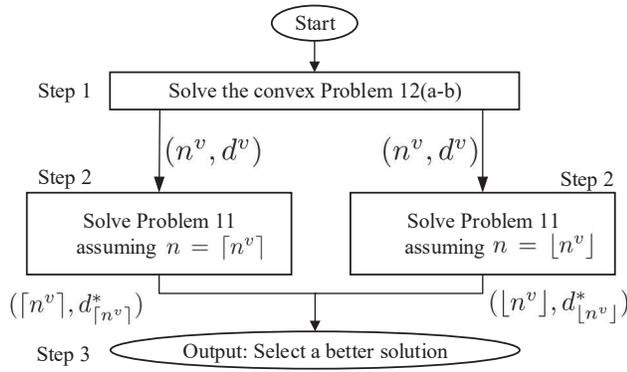


Fig. 6: Optimal Solution Algorithm Flowchart

*Proof.* We first prove that given any  $n$ ,  $s(n, d)$  is concave in  $d$  for  $d \in \{d | \frac{1}{\xi}(e^{-\beta d} - d\beta e^{-\beta d}) - p_e \geq 0\}$ . For any  $d$  such that  $\frac{1}{\xi}(e^{-\beta d} - d\beta e^{-\beta d}) - p_e \geq 0$ ,  $\frac{de^{-\beta d}}{\xi} - dp_e$  is a positive increasing concave function in  $d$ . Meanwhile, it can be seen from (5) that  $P_{\pi_n}(d)$  is a positive decreasing concave function in  $d$ . Therefore, the product  $P_{\pi_n}(d) \left(\frac{de^{-\beta d}}{\xi} - dp_e\right)$  is concave in  $d$ . According to Theorem 1, we have  $\forall n \geq m$ ,  $\frac{\partial^2 \omega_{\pi_n}(d)}{\partial d^2} > 0$ , and  $\frac{\partial \omega_{\pi_n}(d)}{\partial d} > 0$ . This, together with the fact that  $h(\omega)$  is a non-decreasing convex function, implies that  $-h(\omega_{\pi_n}(d))$  is also concave in  $d$ . Hence,  $s(n, d)$  is concave in  $d$  for  $d \in \{d | \frac{1}{\xi}(e^{-\beta d} - d\beta e^{-\beta d}) - p_e \geq 0\}$ .

We now prove that  $s(n, d)$  is concave in  $d$  at an optimal  $n^*$ . This can be proved by showing that the condition  $\frac{1}{\xi}(e^{-\beta d} - d\beta e^{-\beta d}) - p_e \geq 0$  is satisfied at the optimal solution, which we will show by contradiction. Suppose that  $\frac{1}{\xi}(e^{-\beta d} - d\beta e^{-\beta d}) - p_e < 0$  holds for an optimal solution  $(n^*, d^*)$ . In this case, the objective in (13a) is monotonically decreasing in  $d$ , because the derivative of the first term in (13a) is negative in the domain and the second term in (13a) monotonically decreases with  $d$ . This contradicts with the assumption that  $(n^*, d^*)$  is an optimal solution. Thus  $\frac{1}{\xi}(e^{-\beta d} - d\beta e^{-\beta d}) - p_e \geq 0$  must hold for an optimal solution to Problem (13).  $\square$

With Lemmas 3 and 4, we propose a 3-step optimal solution algorithm to Problem (13) in Fig. 6. To recap, the optimal  $n$  and  $d$  obtained in this section defines the JoAP algorithm that optimizes the charging station operation. In particular, the charging station sets the charging price  $r$  that leads the EVs to request demand  $d$ . According to the current charging load at the station, the  $\pi_n$  admission policy decides whether to admit an EV or not. For those admitted EVs are admitted, the FIFO policy is applied to provide charging service.

## V. SIMULATION RESULTS

In this section, we use real-world data to evaluate the performance of the JoAP scheme and investigate how different system parameters affect the profit and admission performance. All the computations are solved in MATLAB [21] on a computer with an Intel Core i5-4670 3.40 GHz CPU and 8 GB of memory.

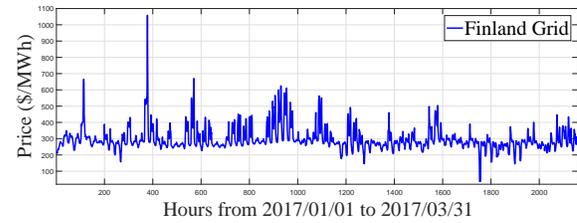


Fig. 7: The hourly prices of Finland Grid from 2017/01/01 to 2017/03/31

## A. Experimental Setup

We base our simulations on the historical hourly electricity prices (Fig. 8a) of Finland Grid in the Nordic electricity market [22]. The data set spans the first 3 months of 2017. All the simulation results are the average performance of 90 days. The EV arrivals follow a Poisson distribution, whose arrival rates in different time periods are listed in Table. I. The settings of the peak hour match with the realistic vehicle trips in NHTS 2009 [23]. Unless specified otherwise, the charging station has  $m = 4$  charging ports with a charging rate  $\alpha = 11.5kW$ . The number of parking lots is 40. All EVs have the same utility parameter  $\beta = 0.05$  and the battery capacity  $\varphi = 100kWh$ .<sup>4</sup> For simplicity, we consider a linear waiting-time penalty  $h(\omega) = c\omega$  [24], where  $c > 0$  denotes the penalty rate. Our proposed JoAP algorithm is flexible enough to adapt its admission control and pricing methods to different EV arrival rates, penalty rates, and electricity prices.

TABLE I: Simulation historical data

Time of Day	08:01-12:00	12:01-16:00	16:01-20:00
$\lambda$ (/minutes)	0.3	0.4	0.4
$\bar{p}_e$ (\$/MWh)	330.2	309.3	335.2
Time of Day	20:01-24:00	00:01-04:00	04:01-08:00
$\lambda$ (/minutes)	0.4	0.3	0.1
$\bar{p}_e$ (\$/MWh)	273.2	244.9	293.2

For performance comparison, we consider the following two benchmark algorithms:

- 1) Queue-length based admission (QBA): An EV is admitted into the system only when the number of EVs already admitted is below a threshold. For our simulations, we set the threshold to the total number of parking lots in the charging station. Such an admission scheme has been widely used in current practice (e.g., California Plug-In Electric Vehicle Collaborative<sup>5</sup>).
- 2) Greedy admission: An EV is admitted if and only if doing so increases the system profit in the short-run (without considering future EV arrivals) [8].

<sup>4</sup>The battery specifications follow the latest information from the Tesla website: <https://www.tesla.com/models>.

<sup>5</sup><http://www.pevcollaborative.org/workplace-charging>

### B. Average Profit Evaluation

In Fig. 8, we compare the average profit per hour achieved by the three schemes under two different waiting-time penalty rates:  $c = \$1/\text{min}$  and  $c = \$0.4/\text{min}$ . For each time period listed in Table I (scenarios), we simulate 1000 independent arrival processes  $\mathcal{V}$  and plot the average profit performance.

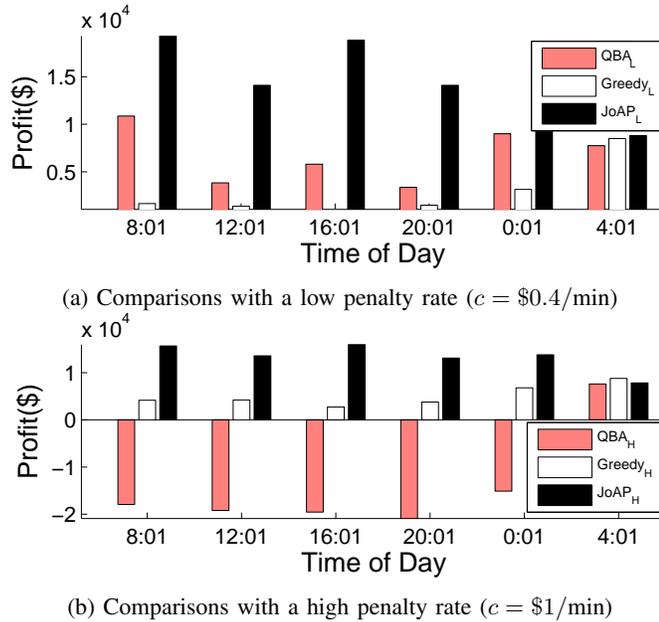


Fig. 8: Comparisons of the achieved profit of different methods in different time of a day

We first compare the average profit of the entire day of three schemes. Fig. 8 shows that JoAP greatly outperforms the two benchmark schemes. The average profit over the whole day is 330% and 531% higher than that of the Greedy admission scheme when the waiting-time penalty is low ( $c = \$0.4/\text{min}$ ) and high ( $c = \$1/\text{min}$ ), respectively. On the other hand, the widely used QBA scheme only achieves 44% of JoAP's average profit when waiting-time penalty is low, and a negative profit when waiting-time penalty is high.

Now we investigate the performance of the three schemes in different scenarios. During low-traffic period, e.g., from 4:01 to 8:00, the advantage of JoAP is not obvious. It only achieves 0.5% and 2% higher profit than the Greedy algorithm under low and high waiting-time penalty rates, respectively. The advantage is more evident under heavy traffics, e.g., 12:01 to 24:00. Under the same traffic intensity, the advantage of JoAP over the Greedy algorithm increases when  $p_e$  increases. This is because the admission probability decreases rapidly when  $p_e$  increases. On the other hand, the advantage of JoAP over QBA decreases when  $p_e$  increases. This is the profit of QBA is dominated by the delay penalty, and therefore is less sensitive to the increase of electricity price  $p_e$ .

It can be seen that the conventional QBA scheme performs very poorly with negative profit when the waiting-time penalty is high. In the events of bulky arrivals, the QBA scheme admits all EVs until there is no available parking lot and denies all the EVs that arrive later. This leads to heavy delay penalty

for admitted EVs and high rejection rate for incoming EVs as well. The Greedy admission scheme has a positive but low profit due to its inability to balance the charging schedule for the current and future EV arrivals. In fact, the Greedy admission scheme always denies some EVs even under very light EV arrival traffic. In contrast, JoAP admits a proper number of EVs by jointly considering the EVs being served and the possible arrivals in the future, thus achieving a much higher profit than the two benchmark algorithms.

### C. Admission Probability Evaluation

In this subsection, we show that the average admission probability of JoAP scheme is comparable with that of the conventional QBA scheme. Fig. 9 compares the average admission probability of JoAP algorithm and the benchmarks under different penalty rates. Overall, the QBA scheme achieves the highest admission probability, i.e., 86%, as it rejects an EV only when the parking lots are full. However, in the some periods with moderate arrival rates, e.g., 8:01 to 12:00, the admission probability of the QBA scheme falls below JoAP as it is oblivious to the possible future arrivals. The overall admission probability of the Greedy admission algorithm is the lowest, i.e., 70% and 69% in the light-penalty-rate and high-penalty rate cases, respectively. JoAP algorithm has an admission probability 85% and 80% in the light-penalty-rate and high-penalty rate cases.

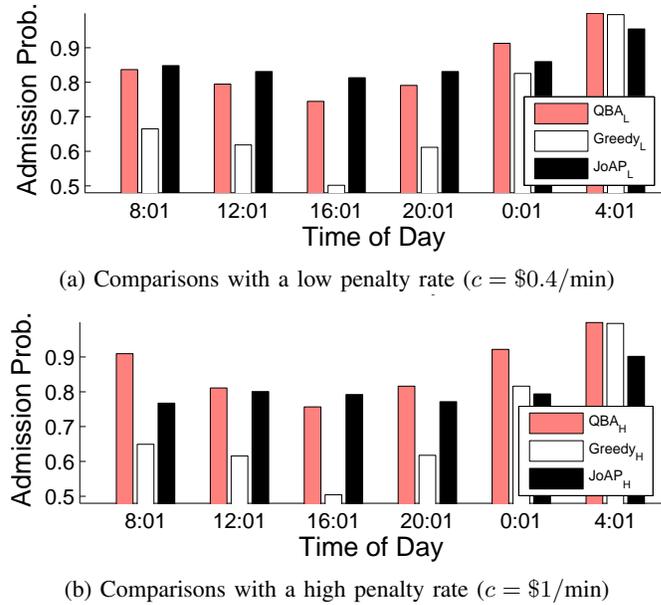
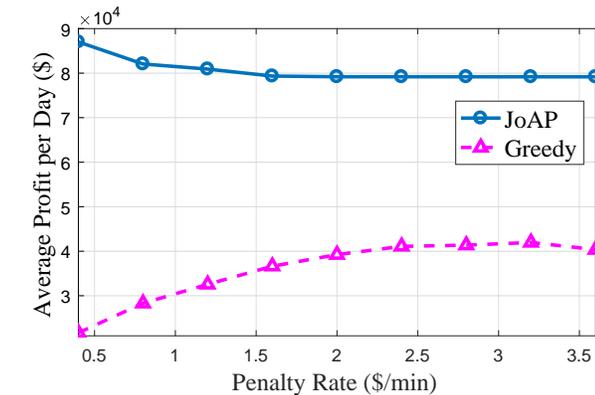


Fig. 9: Comparisons of the admission probability of different methods in different time of a day

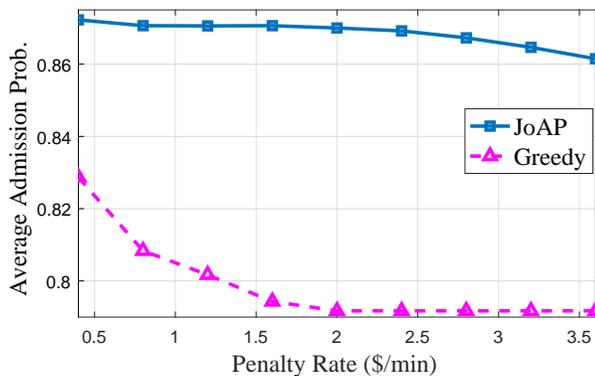
### D. Impact of $c$

We now turn to investigate how different waiting penalty rates impact the admission and pricing. We consider only the Greedy method and JoAP in the following simulations, because the penalty rate has no influence on the admission

policy of the conventional QBA scheme. For each  $c$ , we simulate 100 independent arrival processes  $\mathcal{V}$  for each day. Taking average over 90 days, we plot the average daily profit and admission probability versus the penalty rate  $c$  in Fig. 10. On one hand, we observe that the profit and the admission probability of JoAP decrease slightly, i.e., by 8.9% and 1.3%, when increasing the penalty rate from 0.4 to 3.6 \$/min. On the other hand, we observe that the profit of the Greedy method increases by 98.5% when increasing the penalty rate from 0.4 to 3.2 \$/min and decreases by 6.6% when increasing the penalty rate from 3.2 to 3.6 \$/min. More importantly, we also observe that the admission probability of the Greedy method decreases rapidly when the penalty rate is increased from 0.4 to 2 \$/min. Overall, the average profit of JoAP is 118% higher than that of the Greedy admission scheme and the profit and admission probability performance varies slightly when the penalty rate changes. This implies that JoAP can achieve a good balance between high admission probability and high profit.



(a) The achieved average profit comparison



(b) The achieved average admission probability comparison

Fig. 10: The achieved average profit and admission probability comparison versus  $c$

### E. Impact of $\tau$

We have considered a fixed  $\tau$  in the theoretical analysis in Section III.B. In Fig. 11, we numerically evaluate the performance gain if we optimize the value of  $\tau$ , and comparing with the case of using a fixed value of  $\tau = 1.01$ . For each

$(c, \lambda, p_e)$ , we simulate 100 independent arrival processes  $\mathcal{V}$  and plot the average profit performances with  $\tau = 1.01$  and the optimized  $\tau$ . Averaging over all scenario, *optimizing over  $\tau$  increases the profit over fixing  $\tau = 1.01$  only by 5.9%*. Therefore, we can focus on the optimizing of  $n$  in practice.

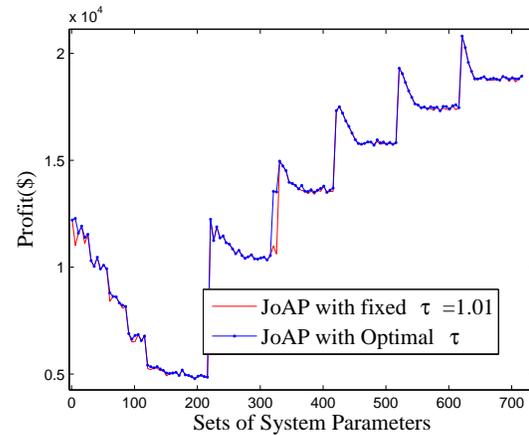


Fig. 11: The achieved average profit with  $\tau = 1.01$  and the optimized  $\tau$  comparison versus  $(c, \lambda, p_e)$

## VI. CONCLUSIONS

In this paper, we proposed a novel joint admission and pricing (JoAP) mechanism for a EV charging station to maximize his profit. In contrast to existing EV charging operation schemes, the JoAP scheme applies a multi-sub-process admission control capable of balancing between the system admission rate and the EVs' QoS requirements according to the EV arrival rate, the electricity price, and the delay penalty. We introduced a tandem queueing model to analyze the joint admission control and scheduling process, and proposed an efficient algorithm to compute the optimal solution. Simulation results showed that JoAP can effectively increase the charging station's profit while providing good QoS guarantees to the EV users.

In our future study, we plan to extend this work to the more general case with heterogeneous EVs. We will further consider how the integration of renewable and distributed energy generations will impact the admission control and efficiency of the charging station.

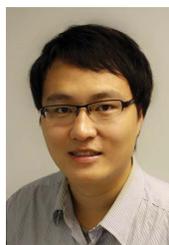
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**Shuoyao Wang** received the B.Eng. degree (with first class Hons.) in information engineering from The Chinese University of Hong Kong, Hong Kong, in 2013. He is currently pursuing the Ph.D degree at The Chinese University of Hong Kong, Hong Kong. His research interests include optimization theory, queueing analysis, dynamic programming, and reinforcement learning algorithm in EV charging scheduling and pricing, and electricity market operation.



**Suzhi Bi** (M'13) received the B.Eng. degree in communications engineering from Zhejiang University, Hangzhou, China, in 2009, and the Ph.D. degree in information engineering from The Chinese University of Hong Kong, Hong Kong, in 2013. He was a Research Engineer Intern with the Institute for Infocomm Research, Singapore, and a Visiting Student with the EDGE Laboratory, Princeton University, Princeton, NJ, USA, in 2010 and in 2012, respectively. From 2013 to 2015, he was a Research Fellow with the Department of ECE, National University of Singapore, Singapore. He is currently an Associate Professor with the College of Information Engineering, Shenzhen University, Shenzhen, China. His research interests include wireless information and power transfer, wire-less resource allocation and medium access control, and smart power grid communications. He was a co-recipient of the IEEE SmartGridComm Best Paper Award in 2013. He is currently an Associate Editor of the IEEE ACCESS.



**Yingjun Angela Zhang** (S'00–M'05–SM'11) received her PhD degree in Electrical and Electronic Engineering from the Hong Kong University of Science and Technology, Hong Kong in 2004. Since 2005, she has been with Department of Information Engineering, The Chinese University of Hong Kong, where she is currently an Associate Professor. Her research interests include mainly wireless communications systems and smart power systems, in particular optimization techniques for such systems. She is an Executive Editor of the IEEE Transactions on Wireless Communications. She is also an Associate Editor of the IEEE Transactions on Communications. Previously, she served many years as an Associate Editor of the IEEE Transactions on Wireless Communications, Security and Communications Networks (Wiley), and a Feature Topic in the IEEE Communications Magazine. She has served on the organizing committee of major IEEE conferences including ICC, GLOBECOM, SmartgridComm, VTC, CCNC, ICC, MASS, etc.. She is now the Chair of IEEE ComSoc Emerging Technical Committee on Smart Grid. She was a Co-Chair of the IEEE ComSoc Multimedia Communications Technical Committee and the IEEE Communication Society GOLD Coordinator. She was the co-recipient of the 2014 IEEE ComSoc APB Outstanding Paper Award, the 2013 IEEE SmartgridComm Best Paper Award, and the 2011 IEEE Marconi Prize Paper Award on Wireless Communications. She was the recipient of the Young Researcher Award from the Chinese University of Hong Kong in 2011. As the only winner from engineering science, she has won the Hong Kong Young Scientist Award 2006, conferred by the Hong Kong Institution of Science. Dr. Zhang is a Fellow of IET and a Distinguished Lecturer of IEEE ComSoc.



**Jianwei Huang** (F'16) is a Professor in the Department of Information Engineering at the Chinese University of Hong Kong. He is the co-author of 9 Best Paper Awards, including IEEE Marconi Prize Paper Award in Wireless Communications 2011. He has co-authored six books, including the textbook on "Wireless Network Pricing. He has served as the Chair of IEEE TCCN and MMTC. He is an IEEE ComSoc Distinguished Lecturer and a Clarivate Analytics Highly Cited Researcher.